Entities without identity: a semantical dilemma

Abstract

It has been suggested that puzzles in the interpretation of quantum mechanics motivate consideration of entities that are numerically distinct but do not stand in a relation of identity with themselves or non-identity with others. Quite apart from metaphysical concerns, I argue that talk about such entities is either meaningless or not about such entities. It is meaningless insofar as we attempt to take the foregoing characterization literally. It is meaningful, however, if talk about entities without identity is taken as elliptical for either nominal or predicative use of a special class of mass terms.

Keywords: non-individual, identity, cardinality, quasi-set theory, mass term, count term, particle individuality, indiscernible

1 Introduction

It is something of a truism that we ought not to read our metaphysics off the structure of our language. But it is also a truism that any metaphysics we might know and describe must be known and described through the medium of language. The way in which meanings attach to words must therefore circumscribe, however loosely, the possible metaphysical theories we are capable of articulating. There are bounds to what can be meaningfully asserted. To stray outside these bounds is to utter nonsense.

There is reason to worry that a prominent metaphysical project in the philosophy of physics has exceeded these bounds. Vexed by puzzles in the interpretation of quantum mechanics, a number of philosophers have urged consideration of entities without identity. This, I suggest, is unhelpful. Either talk about such entities is nonsense, or it is not *about* such entities.

The first horn of this dilemma has been noted before. But prior formulations of the entities-without-identity-are-nonsense objection all suffer from one or more ailments. Each was presented in a much weaker guise than it might have been, or has gone stale in the face of more than two decades of work purporting to make sense of such talk, or has been misconstrued as a metaphysical thesis. The first half of this paper reinvigorates the nonsense objection by repairing these deficiencies. The second horn of my dilemma is novel. It's true that there exist a number of more or less developed interpretations of quantum mechanics that do without the precept of particles as discrete objects and thus, as we'll see below, moot the motivation for considering exotic alternative. But curiously, no one has yet undertaken the exercise of examining the semantic behavior of quantum terms independently of any metaphysical proposal. For the first time then, I sketch a semantics-first approach: ascertain how the truth conditions for uncontroversial propositions of quantum mechanics behave, determine which, if any, established semantic categories these claims fall into, and only secondarily worry about how the world must be if claims with this semantic character are to successfully refer. This task consumes the second half of the paper.

2 Why talk about entities without identity?

2.1 The quantum origins of a radical proposal

The impetus for talk about putative entities without identity is quantum mechanics. The reasons are well-documented elsewhere, but to give a sense for how such arguments go, I'll sketch a standard trope in the literature that draws upon features of quantum statistical mechanics.[1] Ignoring some technical details of quantum state representations, the argument runs like this. Take it as a premise that all distinct (if not qualitatively distinguishable) configurations of particles and properties are equally probable. We might say that each distinct state receives an equal portion of probability. Consider, for example, the case in which we have just two particles, call them p_1 and p_2, and two distinct bundles of properties that may be predicated of each, call them M_1 and M_2. If we denote the proposition that p_i possesses properties M_j by $M_j(p_i)$, then classically there are four distinct states in which we might find such a system:

1. $M_1(p_1) \wedge M_1(p_2)$,
2. $M_1(p_1) \wedge M_2(p_2)$,
3. $M_2(p_1) \wedge M_1(p_2)$, and
4. $M_2(p_1) \wedge M_2(p_2)$.

The probability of finding one particle with M_1 and one with M_2, irrespective of which is which, is given a portion of probability twice as large as finding both particles to possess M_1 (1/2 versus 1/4). This simply reflects the fact that two distinct states match the former description. According to QM, however, states in which one particle has

[1] For a thorough discussion of the role of quantum mechanics in provoking consideration of entities without identity, see (French and Krause 2006). For the specific argument discussed here, see also (Post 1963; Reichenbach and Reichenbach 1999; French 2000).

properties M_1 and the other has M_2 must be granted a portion of probability as though there is but one such state.[2] In other words, the two possibilities (2) and (3) are treated as one. Thus, we have the elements of a modus tollens. If (2) and (3) were distinct states of the world, then each state would have to be assigned a unit weight relative to the other possibilities. Quantum statistical mechanics tells us that in fact the two states *combined* receive a unit of probability. Therefore, quantum state representations with particle names permuted do *not* represent distinct states. This is supposed to suggest that in the case of quantum particles, "labels are otiose" (French 1998, p95).

There are many difficulties with the argument sketched above, and I haven't the space to give the technical issues their due. However, let us take the argument at face value and consider the novel sort of entity it suggests. According to the principal proponents of this view, these are entities that are numerically distinct and yet fail to stand in relations of self-identity. In other words, there is no relation that holds only between an instance of this new sort of entity and itself. As French and Krause (2006) are fond of putting it, they differ *solo numero*. If quantum particles are such entities, this would explain their strange statistics. For there to be a difference between $M_1(p_1)$ and $M_1(p_2)$, it must be the case that the subscripts label distinct particles. But if that were true, the particles would stand in a relation that acts like self-identity. Specifically, particle 1 would be that which uniquely bears the label p_1. Since the entities we've postulated cannot stand in such relations, it must be that they cannot be uniquely referenced. In that case, one cannot assert a distinction between $M_1(p_1)$ and $M_1(p_2)$. Nonetheless, there are determinately many particles in a collection. For instance, there are determinately many hydrogen atoms in a given flask. The posit of entities without identity handily accounts for these quantum facts.

[2] Roughly speaking, there are actually two possibilities corresponding to two basic kinds of particle. For bosons, the states in which one particle possesses M_1 and the other M_2 would receive a combined probability of 1/3 as would each of the states $M_1(p_1) \wedge M_1(p_2)$ and $M_2(p_1) \wedge M_2(p_2)$. For fermions, the only possible state is that in which one particle possesses M_1 and the other M_2.

Of course, that's not enough to force one to abandon particle identity. Rather, the consensus seems to be that we face a sort of underdetermination with respect to a pair of metaphysical packages – one which clings to primitive identities for quantum particles with all of the difficulties this entails for interpretation and another which embraces the strange possibility of entities without identity (French and Krause 2006). It is not my aim to weigh the metaphysical merits of these competing views, but rather to scrutinize the semantic coherence of one of these ostensible alternatives. Expressing *solo numero* difference is – if not impossible – far more difficult than it seems.

2.2 A name for the nameless

But before trying to make this deeper semantic point, I have to dispense with a superficial language problem: what to call the putative entities without identity? In an influential book, Steven French and Décio Krause introduced the term "non-individual" to designate entities for which "the relation of self-identity $a = a$ does not made sense" (French and Krause 2006, p248). These entities are neither self-identical nor non-identical to other entities. The relation of identity simply doesn't apply (see,e.g., Arenhart and Krause 2014). This suggests that I should use the term 'non-individual'. However, in an equally influential series of papers, Simon Saunders and F. A. Muller define "individual" (and by extension, "non-individual") in terms of discernibility (Muller and Saunders 2008; Muller and Seevinck 2009; Muller 2015). Their intent was to bring some clarity to debates over the status of Leibniz's Principle of the Identity of Indiscernibles (PII) in light of quantum physics. This principle says roughly that indiscernible things are identical. But what is meant by indiscernible? To examine the space of possibilities, Muller and Saunders provide a sharp logical hierarchy of types of entity. The most inclusive category in this scheme is the *formal object*. These are "values of variables bound by quantification and subject to predicative identity-criteria, that can in principle be described in elementary predicative formal languages, incorporating elementary predicate logic" (Muller and Saunders 2008, p503). The formal objects in a set are said to be *absolutely discernible* if and only if each has some permissible property that all the others lack. For our purposes, it doesn't matter much what the "permissible" properties are, except to note that they do not include predicates containing identity (Muller 2015). Objects in a set are *relationally discernible* if and only if for every object there is a relation that discerns it from every other object. An object is *indiscernible* if it neither absolutely nor relationally discernible. The term *individual* is reserved for those objects that are absolutely discernible. Thus, in this scheme, *non-individuals* are entities that are not absolutely discernible. This leaves open the possibility – which they defend as actuality – that there exist objects which are non-individuals but relationally discernible.

The hierarchy of Muller and Saunders is all well and good for clarifying the positions one may take with regard to PII, but it leaves us in a bind with respect to the subject of this essay. The problem is that "non-individual" means something different for the proponents of entities-with-no-identity than it does for Muller and Saunders. It may seem on the face of it that we could simply embrace the newly precisionized term "indiscernible". Indeed, proponents of the old non-individuals view invite this reading when they talk about collections of indiscernible non-individuals that are nonetheless not identical with one another (Arenhart and Krause 2014, p7). But this is misleading. If they meant simply that non-individuals are entities for which even the weakest, non-trivial form of PII is false, then they would indeed be indiscernibles in the sense of Muller and Saunders. But they do not mean merely that there can be some pair of entities that have all their permissible properties in common and yet for which it is false that $a = b$. They mean also that it is false that $a \neq b$. Of course, this makes no sense in classical logic, but that's the point. Identity is not supposed to apply to this sort of non-individual. It's perhaps more accurate to say that $a = b$ and $a \neq b$ have no truth values (which is what French and Krause say when they're being more careful). That means that what French and Krause called a non-individual is not even a formal object in the terminology of Muller and Saunders. In part, this is because they are supposed to be objects that cannot bear labels, and consequently, they are not supposed to be "subject to predicative identity criteria."

I see no choice but to drop the now ambiguous term "non-individual". But what shall we use instead? One might consider falling back on Hermann Weyl's term, "effective aggregate" to refer to collections of entities without identity (Weyl 1949). But French and Krause (2006) explicitly reject Weyl's approach to formally representing what they mean by non-individuality (they say Weyl merely "mimics" it). So it seems we need a new term, at least for present purposes. I propose *anonymere* or "nameless part" to capture the idea that the putative entities can form aggregates or collections, but that no member of such is addressable in principle by a proper name or label. Anonymeres, then, are entities for which the relation of identity simply fails to apply, yet which nonetheless exist in definite numbers, and "anonymerity" is the property of being an anonymere.

3 The problem of numerosity

3.1 Running afoul of a conceptual truth

What can it mean for anonymeres to differ *solo numero*? It's easier to say what this cannot mean. If 'numerical distinctness' is understood in anything like the usual way, then anonymeres cannot be numerically distinct and yet fail to be identifiable. In the standard set-theoretic definition, cardinality essentially involves a notion of mapping or correspondence that is conceptually equivalent to labeling. Specifically, the cardinal associated with a set A (intuitively, the number of things in A) is the smallest ordinal number n such that there is a bijection from the elements of A to the elements of n. To put it more plainly, two sets are the same size as one another if their elements can be brought into one-to-one correspondence, that is, if the elements of one can be used to uniquely label elements of the other. Ordinals are just specially constructed sets whose elements are rigidly ordered (i.e., there is a binary relation, $<$, such that for every pair of elements a and b, either $a < b$ or $b < a$). The ordinal number 3, for instance, is the set $\{0, 1, 2\}$. The cardinality of a set is the smallest ordinal of the same size. Put yet another way, the cardinality of a collection is what we get by a generalized counting procedure in the intuitive sense of counting. After all, counting is really just a sort of indexing by which we point at distinct things and label them by saying "one", "two", "three", etc.[3]

The relation between identity and cardinality is not a metaphysical fact, but rather a semantic one. I do not mean to suggest that there is some metaphysically necessary association between identity and cardinality. Rather, I am claiming that what it *means* for entities in a collection to be numerically distinct is for the collection to possess a cardinality greater than one. And part of what it *means* for a collection to possess a definite cardinality — on any standard account of cardinality — is for the entities to be identical with themselves and no others in the collection.

This point has been made before, or at least previous arguments may be read consistently in this way. For instance, Lavine (1991, p260) argued briefly that anonymerity is incoherent in that it cannot explain, "for example, what it is for there to be two distinct photons in the box – after all, being 'distinct' means 'having different identities'." And

[3]The cardinality of 'uncountable' sets – such as the set of real numbers – obviously cannot be expressed via an ordinal. Nonetheless, the notion of equinumerosity or equal cardinality that sustains talk of such extended notions of size is still dependent upon the notion of a bijection and thus of the identity of the elements of the set. The same arguments about the connection between cardinality and identity thus apply. But as long as we're talking about particles in non-relativistic quantum mechanics, there are at most countably many, and so we needn't complicate the discussion by considering collections or sets that are uncountable.

it seems to me that in (Jantzen 2011b), I make the conceptual link between cardinality and identity clear as part of a critique of 'ontic structural realism' (see, e.g., Ladyman, Ross, and John Collier 2007). However, my thesis about the conceptual connection between cardinality and identity has occasionally been read as a metaphysical claim, perhaps because the primary target of my criticism in that paper is a metaphysical theory. For example, Arenhart (2012, p807) takes the claim to be that the "traditional definition of cardinality and counting is so simple and basic that it has more rights over other metaphysical claims." Given such misapprehensions in the past, I want to avoid misinterpretation of my current claim: I am asserting that identity and cardinality are tied together as a matter of meaning irrespective of metaphysics. And on this point, there is much more to be said than appears in the existing literature.

3.2 The relevant notion of identity

First, it is important to note that cardinality presupposes only a relatively weak concept of identity with respect to a class, namely that class which is to have its cardinality assessed. Consider a collection of things, S. To say that S has a definite cardinality is in part to say that there is some relation R on the members of S so that for every $x \in S$, the relation holds only between x and itself. However, it is not necessary that the relation R has this feature for every entity whatsoever, or even every entity of this or that kind to which the members of S belong. For example, if one wants to determine how many objects are in a visual scene, it is sufficient to have an "is the same as" relation on visual objects such that each object satisfies the relation with respect to itself but not with respect to any other object in that scene. However, that relation need not discriminate amongst objects in other visual scenes. It may, for instance, distinguish an apple from an orange in a scene containing at most one of each fruit, but fail to discriminate with respect to apples in other scenes.[4] One might be inclined to deny such a relation the title of "identity" and I would have no quarrel. I would be happy to refer to such relations as relations of "contextual identity." For present purposes, I need not take a stand on this issue one way or the other. The point is that cardinality presupposes only a weak notion of contextual identity, but the notion of an anonymere, if it is to do the work it was created to do, must deny even this much.

This point about the required identity concept is worth belaboring since it is generally overlooked. To give one example, Domenech and Holik (2007) offer what they take to be a definition of cardinality worthy of the name and suitable for the case of a collection of anonymeres. Suppose we have such a collection, X. Informally speaking, their proposal depends upon the notion of a 'quasi-singleton' defined with respect to X. For some $x \in X$, the quasi-singleton $\langle x \rangle$ is a collection whose only sub-collections are the empty collection or itself.[5] This is certainly a feature one would expect for a collection that contains just one thing. The cardinality of X is then defined by constructing a series of collections, each of which is derived from the previous by removal of a quasi-singleton. The series terminates (if at all) when there is nothing left to remove. Essentially, the cardinality of X is given by the length of the chain of derived collections.

This procedure is ingenious and perfectly consistent. However, it fails at its stated aim. As I prove in (Jantzen 2011b), the notion of a quasi-singleton as Domenech and Holik define it provides an identity relation on the elements of X *relative to the collection being counted*. In other words, relative to the collection X and all collections that may be constructed from X, membership in the same quasi-singleton acts as an identity relation. It is no surprise then that we can sensibly count using this procedure – we

[4] Of course, some other relation must function as identity if we are to count fruit across multiple scenes.

[5] The proposal of (Domenech and Holik 2007) is spelled out in terms of quasi-set theory discussed below. Thus, the technical definitions involve quasi-sets. However, to efficiently convey the gist of the proposal, I am using the neutral term "collection" instead.

implicitly appeal to that identity relation. Arenhart (2012) has attempted to rebuff my critique by pointing out, quite rightly, that the putative identity relation is not a first-order identity relation on the whole universe of anonymeres and collections of them, supposing there are others in it besides those of X. My point is simply that this is irrelevant. The relation in question acts like an identity relation for the collection one is counting. What's supposed to make anonymeres special is that a collection of them possesses a distinct cardinality without any relation *relative to that collection* serving to distinguish them one from another. We wouldn't worry whether electrons in a collection are in some global sense *really* anonymeres if it were always the case that in any given collection of electrons, there is a property that distinguishes them from each other. Then we could simply count the things so discriminated in the old-fashioned way of counting. There would be nothing to motivate talk about entities without identity.[6]

Again, I want to stress that I am advancing a semantic claim, not a metaphysical theory of identity. This semantic claim puts relatively few constraints on the metaphysics. When I claim that the meaning of 'cardinality' is dependent upon the meaning of 'identity', I intend to refer to a concept of contextual identity. That concept of contextual identity can be quite thin, involving only sameness and difference. Metaphysically, it may be that identity always requires distinguishability as Leibniz would have it.[7] Perhaps not. Perhaps identity is primitive in some way, independent of all other intrinsic properties as Adams (1979) would have it. Perhaps not. Perhaps there is a unique relation of identity, or perhaps identity resolves into a myriad of relations. But there are some constraints. It is a conceptual truth that some relation with the features of

[6]Bueno (2014) makes a related point. Proponents of anonymerity (e.g., (Domenech and Holik 2007)) often point to a physical phenomenon that ostensibly offers a clear example of counting without identity. Imagine stripping electrons off of a large atom, perhaps by blasting them with electromagnetic radiation of sufficiently short wavelength. As the electrons fly off, they can be detected by, say, their tracks in a cloud chamber. In that way, we could count how many electrons were removed from the atom, even though QM tells us we cannot say which is which. As Bueno points out, however, we cannot say that each track corresponds to a *different* electron unless there is a way to identify them.

[7]See, e.g., (Saunders 2003).

identity must obtain for a collection (relative to that collection) if there is to be a definite number of things in the collection. Of course, there may be no things in the world that answer to our concepts of identity and cardinality, but if there are, then cardinality must depend upon identity in this way.

This conceptual truth – that cardinality presupposes identity – is independent of the status of Leibniz's Principle of the Identity of Indiscernibles (PII) in QM (see, e.g., French and Redhead 1988; Saunders 2003; Caulton and Butterfield 2012). PII is often interpreted as the metaphysical principle that relations of identity can be grounded in facts about qualitative difference. If that's true, then there cannot be a multiplicity of indiscernibles. If it's false, then there can be. Either way, the PII speaks neither for nor against the claim that cardinality presupposes identity. Similarly, the question at issue is largely orthogonal to concerns over the 'weak discernibility' of quantum particles. Even if all quantum particles, as Muller and Seevinck (2009) argue, are weakly discernible in that some irreflexive relation obtains on any given collection of particles, this does nothing to weaken the semantic bond between identity and cardinality. To the contrary, as (French and Krause 2006) point out, the very notion of an irreflexive relation would seem to presuppose an identity relation on the particles themselves, at least conceptually if not materially. I suggest that's why it makes sense to assert that there are a determinate number of particles that are only weakly discernible.

3.3 The conceptual link between identity and cardinality

It might appear that I've oversold the conceptual connection between cardinality and identity. After all, those who wish to deny this conceptual dependence are able to point to arguments in the philosophical literature that suggest the conceptual separability of identity and cardinality. Specifically, they point to E. J. Lowe's (1998) arguments for the relative conceptual and metaphysical independence of identity and cardinality.[8] Lowe suggests that it's at least conceivable for there to be entities with determinate cardinality but no identity.[9] Unfortunately, his argument rests on an appeal to the very same suspect analysis of quantum mechanics described above. Specifically, Lowe understands the physics to imply that, when found in entangled states, quantum particles such as electrons have no definite identity as there is no fact of the matter which is which. His example is the capture and release of an extra electron by a helium ion, a physical system for which there is supposed to be no fact of the matter which of the electrons arrived last. Lowe gives no motivation for interpreting the physics in this way.[10] From his references, it appears that this interpretation derives largely from the work of the proponents of anonymerity.[11] Wherever the interpretation comes from, it is only because Lowe is swayed by the assertions that quantum particles have definite cardinality but no identity (at least some of the time) that he thinks it possible to have one without the other. In other words, his argument for conceptual separability is a proof by example: quantum particles are supposed to be entities with the requisite features. As a defense of the anonymere thesis this argument would be trivially circular, with the proponents of anonymerity appealing to Lowe's argument which in turn rests on the assertion of the existence of anonymeres. Thus it seems that outside of the motivations provided for adopting the so-called 'Received View' of quantum statistics (French and Krause 2006; Arenhart 2015) there are no independent arguments for cardinality without identity on offer.

[8]Lowe uses the term "countability" rather than cardinality. This invites confusion in his discussions of continuously divisible substances, given the technical sense of countability.

[9]Lowe also claims, perhaps more surprisingly, that it's possible for there to exist entities with identity but no determinate cardinality. I have been arguing that a notion of identity is conceptually prior to a notion of cardinality, and so only the first of these possibilities is logically pertinent to my argument.

[10]In fact, I have argued (in (Jantzen 2011a)) that treating, e.g., electrons, as sometimes entangled and sometimes not is empirically adequate under certain conditions, but is simply inconsistent as a matter of interpretation.

[11]Lowe specifically cites (French and Redhead 1988; Redhead and Teller 1992), which defend the conceptual coherence of entities without identity in the interpretation of quantum physics.

If anything, I've probably underestimated the conceptual connection between identity and cardinality. The connection is essential to the first attempts at explicating the notion of number, that is, of "how many". The definition I alluded to above – equinumerosity means one-one correspondence – is essentially Cantor's. Much the same is there in Frege's *Foundations of Arithmetic* where he dubs the core idea "Hume's Principle." He's referring, of course, to Hume's remarks in Section 1.3.1 of the *Treatise*: "When two numbers are so combin'd, as that the one has always an unite answering to every unite of the other, we pronounce them equal"(2004, p51). Interesting concerns have been raised about this ostensible principle (Heck 2000), but the issues boil down to a debate as to whether counting is conceptually prior to equinumerosity or vice versa. What's not at issue is the presumption of identity for the things tallied up by a notion of cardinal number. Aside from a deep mathematical and philosophical tradition that understands cardinality, at least indirectly, in terms of identity, there is reason to suppose that such a notion of cardinal number is deeply embedded in our cognition. Hume's Principle may be austere, but it reflects a concept of 'numerosity' manifest in human infants, non-human vertebrates like chimpanzees, and even invertebrates such as honeybees (Brannon and Roitman 2003; Beran 2012; Dacke and Srinivasan 2008). Of course, the claim that, e.g., honeybees possess the concept of numerosity is predicated on a particular operationalization of the concept; you can only measure what you test for. But that's the point. The fact that the experiments were designed to look for a concept of cardinality compatible with Hume's Principle suggests that, whatever other concepts they may possess, people and some nonhuman animals share a concept well captured by classical definitions of cardinality. If anything has a claim to a deeply intuitive idea, it's the notion of cardinality that depends upon identity.

The upshot of the foregoing considerations is that, while there is some uncertainty concerning the relative conceptual dependence of notions of equinumerosity and counting, and some dispute as to the right way (if there is indeed a uniquely best way) to formalize the notion of cardinality, there is broad agreement on one thing: to assert that there are n things in a collection is to assert that there is a relation of identity amongst those things such that each is (at least contextually) identical to itself and not any of the others in that collection. To say that the members of a collection are anonymeres, is to deny even a weak, contextual notion of identity. Note that this is a conceptual fact, not tied to any particular formalization of a physical theory. Lam (2014) rebuts those who deny the coherence of anonymerity by suggesting that arguments like mine (Jantzen 2011b) confuse the theory with its formal representation. Anonymerity is supposed to be a feature of things, not of the set theory or other formal apparatus used to describe them. I agree with this last point but, from the semantic perspective at least, it's irrelevant. Regardless of formalization, if it is true that a theory is *about* anonymeres, then the physical theory itself requires us to make sense of that aboutness. Yet we cannot consistently express the idea that a determinate number of things are anonymeres. Given the intended features of anonymeres, the notions of 'cardinality' and 'numerical distinctness' must mean something else when invoked to define or describe them. So what is meant?

4 Quasi-sets as quasi-solutions

4.1 Characterizing a concept by axiomatization

When intensional definitions that employ established terms in the definiens prove inadequate to convey the sense of a radically new concept, one might instead attempt to establish the precise meaning of a new term by providing an account of the semantic role of that term. A particularly effective tool for doing so is formal axiomatization. Axioms in a formal language are satisfied by a subset of all possible models. Examining those models that satisfy the axioms tells you something about what features of the world (or possible worlds) correspond to an unknown term. In this way, concepts can be defined, or at least circumscribed, by formal axiomatizations. So, for instance, if you wish to understand geometric concepts like 'triangle', you could consider Euclid's axioms. If you want to understand the general notion of distance, you could consider the axioms satisfied by all metrics, and so on. A prominent attempt to do this for anonymeres comes from Decio Krause,[12] who axiomatized a theory of "quasi-sets"(Krause 1992).

The formal theory of quasi-sets, $\mathfrak{Q}$, is presented in a first-order language. It was designed as a conservative extension of ZFU, the Zermelo-Frankel axiomatic theory of sets with *urelemente*. In other words, there is a 'copy' of ZFU within $\mathfrak{Q}$ such that theorems of ZFU are theorems of $\mathfrak{Q}$ as well. What's different about $\mathfrak{Q}$ is the introduction of a second kind of *urelemente* or "atom". In classical set-theory, the atoms are presumed to stand in relations of identity such that each is identical with itself and no other. Krause and French call these "M-atoms." In $\mathfrak{Q}$, these classical M-atoms are complemented by others which can be members of so-called 'quasi-sets', can stand in relations of indistinguishability (e.g., each is indistinguishable from itself), but do not stand in relations of identity. They call these "m-atoms." If x and y are m-atoms, then "$x = y$" is not a well-formed expression of $\mathfrak{Q}$. This reflects the fact that, in the intended interpretation, there is supposed to be no fact of the matter whether one m-atom is identical with another. Of course, the point of the formal theory is to pin down this intended interpretation. The m-atoms are the formal counterparts of anonymeres. By interpreting m-atom terms in $\mathfrak{Q}$, we're supposed to get a grip on what talk about anonymeres refers to. It is in attempting to interpret $\mathfrak{Q}$ however, that we encounter a problem.

[12]The theory was later revised in collaboration with Steven French (French and Krause 2010; French and Krause 2006).

4.2 A general problem for formal theories of entities without identity

The problem is that formal theories in first-order languages like ZFU are standardly interpreted according to Tarskian semantics. Speaking coarsely, an interpretation is a structure that includes the specification of a domain of discourse, D, as well as properties, relations, and functions on that domain. The domain of discourse is understood to be a set of objects (whether mathematical or physical objects). Properties and relations amongst the objects in the domain are defined extensionally, e.g, each binary relation R is presented in the interpretation as a set of ordered pairs $\langle x, y\rangle$ such that x stands in R to y. Sentences in the formal language are interpreted by mapping names and variables to objects in the domain and predicate symbols to properties and relations on D. Sentences in the language are true in a structure (i.e., a particular domain and set of extensionally defined predicates) just if the interpreted sentence is true. If all sentences of the theory are true, the structure is said to be a model of the formal theory.

The theory $\mathfrak{Q}$ does have models in the Tarskian sense. In fact, it can be modeled by the sets of ZF.[13] That's how French and Krause (2006) prove the consistency of their axioms. The problem is that the objects in the domain of discourse must have identity. In part, this is because that is how the elements of classical sets are conceived. But in part the problem is implicit in the general form of Tarskian semantics, whether or not we consider the domain a "set." I said that names and variables "correspond" to or "map" to objects in the domain of discourse. For this to be the case, it must make sense to assert that a particular constant or variable refers — and refers uniquely — to an object. The m-atoms of $\mathfrak{Q}$ – or more accurately, the things to which variables in the theory are supposed to correspond – lack this feature. If $\mathfrak{Q}$ is to be interpreted in terms of anonymeres, there cannot be a mapping or reference relation between an m-atom

[13] That is, by Zermelo-Frankel set theory without *urelemente*, only sets.

symbol or term in the theory and a unique entity. For this reason, the interpretation of $\mathfrak{Q}$ requires a new semantics.

I do not mean merely that it requires us to consider structures other than classical sets as models. The problem is rather deeper. The relation between formal sentences and possible worlds in the Tarskian scheme is one in which symbols label or uniquely and unequivocally denote particular objects. Furthermore, we're supposed to be able to describe properties and relations extensionally with sets of ordered tuples. Neither of these is possible for anonymeres. What we need is a whole new semantics for the formal theory.

4.3 Vicious circularity

In an attempt to find an alternative to Tarskian semantics, Arenhart and Krause (2009) have undertaken the exercise of constructing both the logic underlying $\mathfrak{Q}$ and a formal semantics for the theory in terms of $\mathfrak{Q}$ itself. That is, they have used the theory of quasi-sets to define the language and state the axioms of $\mathfrak{Q}$, and to provide a semantics for interpreting sentences of the theory. As they point out, something similar can be done for classical ZF. Depending on one's purpose, there is nothing illicit about using one and the same language as both the object language and the metalanguage. But if the purpose is to use axioms along with a given formal semantics to understand a new concept, it is problematic to describe the semantics in terms of the very concept we are trying to understand. The question is whether we can use the new notion of a 'model' of the axioms of $\mathfrak{Q}$ to help us understand terms like 'anonymere'.

To see why there might be a problem with this strategy, consider the classical case. Tarskian semantics shares only a handful of notions with set-theory, namely the idea of a set, ordered tuple, and mapping or assignment. These notions all have relatively clear, intuitive meanings independent of any particular formal theory or mathematical exposition. Thus, if one wanted to understand an idea like power set or well-ordering, then one could use Tarskian semantics to investigate models of a collection of axioms without circularity or contradiction. But if one wanted to understand, say, identity there would be trouble. Identity is one of the notions essential to Tarskian semantics, and so looking at models of a theory of identity would teach us nothing about identity. We would have to know what identity means before we could understand what models are in the first place. For understanding anonymerity, quasi-set theory is similarly impotent. We cannot understand anonymerity using a semantics stated in terms of $\mathfrak{Q}$ because the relevant notions for stating the semantics derive from the notion of anonymerity. The portions of quasi-set theory we need in order to understand the proposed semantics involve precisely the notion we are trying to learn about by employing a formal semantics. Just as classical formal logic with Tarskian semantics is not very helpful for teaching us what identity is, quasi-set theory with its mysterious semantics is useless for gaining an understanding of anonymeres. For this purpose, the relation between formal semantics and meta-language is viciously circular.

4.4 Determinate lessons from 'vague identity'

We can perhaps see the problem more sharply by viewing it from a vague perspective. More specifically, the metaphysical literature on ontological vagueness has precipitated proposals for expanded concepts of identity that suffer similar semantic difficulties, and understanding why difficulties arise in this other context may clarify the problem for anonymeres. In the literature to which I'm referring, the principal question is whether vagueness of identity can inhere in the world rather than the language we use to talk about it. In other words, does vagueness arise in the world because the identity relation itself is fuzzy or indeterminate? For example, think of an evaporating puddle that spans the road in the morning but by afternoon results in a pair of puddles, one on either shoulder. At any time during the day, we could point to the water on the western shoulder of the road and call the puddle of which it is a part "West Puddle", and similarly we could identify water on the east side as belonging to "East Puddle". It's clear that, in the morning, the puddles so identified are identical. That is, East Puddle = West Puddle. And it's equally clear that by late afternoon, the puddles so identified are distinct, i.e., East Puddle $\neq$ West Puddle. But for much of the morning it is vague whether or not the puddles are identical. Some take that vagueness to be a feature of the identity relation itself – they claim it is a metaphysical fact that it is indeterminate whether East Puddle = West Puddle.

Unsurprisingly, a few philosophers have balked at the notion of indeterminate identity. Smith (2008) in particular has argued that the idea is simply incoherent, at least as it has been framed thus far. In a nutshell, Smith's argument runs as follows (2008, p8):

1. To make clear sense of something one must (at least) model it set-theoretically.
2. Vague identity cannot be modelled set-theoretically.
3. Therefore we cannot make clear sense of vague identity.

It should be clear already that Smith's argument against vague identity echoes the argument above against the conceptual coherence of anonymerity. But a few clarifications are in order. First, in premise (1), Smith does not have any particular set theory in mind (e.g., ZF, ZFC, non-well-founded theories, etc.), but rather a common core of such theories that he calls 'Chapter One' set theory. At the very least, this includes the notion of a collection of objects and a well-defined notion of membership. Both of these presuppose a determinate relation of identity on the objects. Second, Smith does not intend to suggest that the only possible way to make sense of the syntactic presentation of a theory is through set-theoretic model theory, but rather that this is the default. If something else is required or intended by the originator of a theory, than it is incumbent on her to characterize an alternate semantics. I don't want to endorse this claim in general, but it certainly seems apt in the context of a theory like $\mathfrak{Q}$ that is presented axiomatically in a first-order language. This brings us to the third point about Smith's argument: he explicitly takes it as an indictment of the quasi-set project. Noting that French and Krause provide only a set of axioms and an assurance that they aren't to be interpreted in terms of classical sets, Smith argues that we lack the resources to make the quasi-set proposal intelligible. Specifically, we lack a model theory that can make room for collections of objects without identity.

In response, Krause (2014, p187, fn3) merely brushes off Smith's argument in a footnote:

> I recall here two points: firstly, I am not discussing vague identity. According to me, identity simply has no meaning in the quantum realm, and I try to explore such a view. Secondly, in speaking of vague identity, Smith is no more making reference to identity, but to another concept, perhaps some other congruence relation.

The first of these points is both uncharitable and, ultimately, self-defeating. Though it is accurate in a narrow sense that Krause is not concerned with vague identity, it is uncharitable to presume that Smith misses this fact. Smith's point is that the problem he identifies for collections of things with vague identity pertains as well to Krause's quasi-sets of anonymeres. And the rebuttal is ultimately self-defeating because, as I have suggested above, the claim that identity is *meaningless* rather than vague for 'collections' of anonymeres just makes matters worse. Krause's second point appears to suggest that whatever 'vague identity' may be, it must be some relation distinct from and coexisting with identity. I suspect Smith would be happy to endorse this claim. His whole point is that the phenomena involving so-called vague identity must derive their vagueness from some other relation – it can't be identity.

A second reply to Smith comes from the literature on vague objects (Smith is not the only one to connect the project of quasi-set theory with concerns over vague objects), and takes aim at his criterion for intelligibility. Darby (2014, p100) claims that "...adherence to set theory is an instance of conceptual conservatism that being naturalistically inclined, we should side with Ladyman and Ross in rejecting." This challenge is subsidiary to a general thesis that 'naturalistic metaphysics' ought not be restrained by any philosophically motivated semantic limitations. Darby doesn't explain why adopting a naturalistic bent allows one to transcend the semantic and conceptual limitations of a computationally bounded agent, but merely offers some considerations that are supposed to make us suspicious of any *a priori* limits to intelligibility. To this end, he points out that we philosophers have collectively been forced to change our minds about what is intelligible in the past as, for instance, in the case of non-Euclidean geometries.

I agree with Darby that there are multiple strategies for making a concept intelligible, and I am unsure what the hard limits to human intelligibility are, if indeed there are any. But contrary to Darby, I must agree with Smith that, until we are presented with a concrete alternative, we are forced to accept certain semantic limitations in the short run. Specifically, I have been arguing that we have no way to imbue statements of anonymerity with meaning. Darby thinks that an appropriate alternative semantics may be forthcoming, and points the way toward developing such a thing using the tools of category theory. This is an interesting suggestion, and worth attempting. But until it it is actually done, there is no sense to be made of the axiomatic theory of quasi-sets. A promise to look for a semantics is not a semantics, and while it is difficult to prove that making the notion of anonymerity intelligible is impossible, it is straightforward to show that it is not yet actual.

Though there may not be an impossibility proof, there is good reason to suspect that *no* alternative semantics can succeed at making sense of collections of anonymeres with determinate cardinality. The lesson to be taken from the foregoing exchange over vague identity is that concerns like Smith's strike much deeper when one attempts to consider things for which identity is outright "meaningless". To see this, we need to shift focus from the space of possible "models" we might deploy to the way semantics attaches such models to an axiomatic theory in a formal language, or for that matter, a natural language. The entire project of semantics involves *mapping* linguistic entities to things, whether they be constituents of an abstract model used to make sense of a novel concept or constituents of the world itself. As Speaks (2016, Sec. 2.2.2) aptly puts it, theories of meaning share "the assumption that in giving the content of an expression, we are primarily specifying something about that expression's relation to things in the world which that expression might be used to say things about." This is so whether one adopts a propositional or a Tarskian approach to semantics, and whether one's models are drawn from set-theory or category theory or the world of physical objects. So it looks like just about any theory of meaning that we would recognize as such involves mapping from expressions to the things they are about. And herein lies the problem. If that mapping only connects linguistic entities to ostensible aggregates of anonymeres, then we never have reason to view these as *aggregates*. If the mapping penetrates to the internal structure of the aggregates, then it presumes identity for the components of structure to which it maps. Thus, it's difficult to see what sort of options there are for an alternative theory of meaning that can do without identity for the things that sentences are about. It seems we have to undermine the very notion of providing a meaning in order to provide meaning for quasi-set theory.

5 A semantical dilemma

If we cannot understand anonymeres through the formal axiomatic theory of quasi-sets, how should we understand talk about anonyemeres? I suggest we look again to physics. It was physics that motivated such talk in the first place, specifically puzzles about quantum particles. It is clear that at least some talk about quantum particles is meaningful. Physicists successfully coordinate expectations, agree on logical relations, and by and large agree on truth conditions for claims about systems of quantum particles. If we assume they are successfully talking about anonymeres, what can be said about the meaning of this term? The answer is surprisingly mundane. In those assertions about ostensible particles or systems of particles which undeniably have semantic content, references to particles behave not like count terms as one would expect for either classical particles or anonymeres, but rather like mass terms.

The distinction between mass and count terms is in the first place a syntactic or grammatical one.[14] By "term" I mean to encompass nouns, nouns that have been 'determined' by addition of an article or quantifier, and noun phrases involving adjectival modifications of nouns.[15] Syntactically, there are terms[16] like "child" or "marmot" that can be pluralized (e.g., "children" and "marmots"), preceded by cardinal quantifiers (e.g., "three marmots"), and appear with indefinite determiners (e.g., "a child"). These are, grammatically at least, the *count terms*. On the other hand, there are terms such as "milk" and "honey" that cannot be pluralized, or appear with cardinal or indefinite determiners. Instead – and unlike the count terms – they can appear with indefinite quantifiers like "much" and "little" (e.g., "much honey")(see, e.g., Wisneiwski 2009). These are the grammatical *mass terms*.

[14] According to Gillon (1992), it was first noted as a grammatical phenomenon in English by Jespersen (1909, vol. 2, ch. 5.2).

[15] For a finer-grained classification, see (Pelletier 2009).

[16] I agree with (Koslicki 1999) that the mass/count distinction properly applies to *occurrences* of a term, not the term itself. For ease of exposition, however, I will elide this distinction.

But the mass/count distinction can also be drawn semantically. At least since Quine (1960), it has been common practice to systematically divide terms on the basis of their behavior with respect to reference. The exact best way to characterize this behavior is a fraught question, but we can identify five features that, at least approximately, seem to capture those aspects of the semantic behavior of mass terms that set them apart from count terms (adapted from Pelletier 2009, p128):

(MT1) Mass terms are *true of stuff*.

(MT2) Mass terms are *divisive in their reference* in that they apply to the parts if they apply to the sum of the parts.[17]

(MT3) Mass terms are *cumulative in their reference* in that they apply to the sum of the parts if they apply to the parts.

(MT4) Stuff that mass terms are true of *cannot be counted*.

(MT5) Stuff that mass terms can be true of *can be measured*.

Conditions (MT2) and (MT3) refer implicitly to spatial parts. That is, division and cumulativity are with respect to spatial composition. Thus, for a canonical mass term like water, if the stuff at A is water and P is a spatial part of A, then we can say the stuff at P is water. Similarly, if the stuff in two spatial regions P and Q is water, so is the stuff in the union of the two regions.

Even with this restriction to spatial division in mind, condition (MT2) should almost certainly be weakened to read "mostly divisive" or even "partially divisive". The term

[17] Note that this use of "divisive" is subtly distinct from what Quine (1960) has in mind when he speaks of division of reference. For Quine, mass terms before the copula of a sentence act as singular terms in that they refer to a single collective rather than a multiplicity of things. In this sense, they fail to "divide their reference". On the other hand, Quine notes that, at least to a degree, when predicative use of a mass term is true for a thing (e.g., "The liquid in the glass is water.") it's true for parts of the thing (e.g., "The liquid at the bottom of the glass is water.")

"water" is often offered as a paradigm mass term, but the stuff to which it refers is clearly not indefinitely divisible (eventually, one encounters discrete molecules). Likewise for ordinary mass terms like "furniture" and "clothing". Clearly, "furniture" divides its reference in a way that "tiger" does not. Half a tiger is plausibly not a tiger, but half a roomful of furniture still contains furniture. Nonetheless, there is a limit to the division – half a chair is probably not "furniture", anymore than half a water molecule is "water". Whatever the right way to characterize the divisiveness condition, it's important to recognize that, even for ordinary mass terms, it is often only approximately met.

These two ways of dividing terms – by their grammatical properties and by their behavior with respect to reference – are on the face of it independent of one another. And in fact the two criteria often yield different judgments. In ordinary speech, there is no guarantee that a term which functions syntactically as a mass term refers to something which has the stereotypical features of a continuous substance. For example, in the sentence "Some people like data better than theory" (Pelletier 2009, p127), both 'data' and 'theory' act syntactically like mass terms but are generally considered to refer to discrete entities. However, enough of the terms that behave syntactically like count (or mass) terms also behave semantically like count (or mass) terms to suppose they are aspects of a single phenomenon. I will use the term *canonical count term* to refer to count terms which behave both syntactically and semantically as such, and *canonical mass term* likewise to refer to any mass term that is consistently so in both syntax and semantics.

With this new vocabulary, I can restate the aim of this section: to determine whether we should read assertions about quantum particles as references involving canonical count terms or not. I claim that the pattern of truth values for uncontroversial statements suggests to the contrary that what are syntactically count terms in such assertions actually refer (if they succeed in referring at all) to something that would more aptly be described via the semantics of mass terms. In other words, we can make sense of quantum claims using the traditional semantics applied to canonical cases of mass term reference. Put yet another way, we can rephrase sentences containing syntactic count terms as sentences containing syntactic mass terms, and the truth conditions of these new sentences – applying the semantics of canonical mass terms – is compatible with the truth conditions warranted by the theory of QM.[18]

A couple of caveats are in order. First, quantum mechanics is a mathematical theory, and the components that do the work of representing states of the world are mathematical objects – Hilbert spaces and operators on them. There are no mass terms or count terms in the theory. Instead, what I and the authors I've cited have been focusing on, albeit implicitly to this point, are assertions expressed in natural language that are more or less uncontroversially sanctioned by the theory. These claims are necessarily interpretive, though minimally so. To accept such a statement as a consequence of the theory is not necessarily to endorse a particular interpretation of quantum mechanics beyond the minimum required to extract empirical content. Second, we must distinguish semantic from metaphysical issues. From the semantic perspective, the question at issue is whether or not quantum predicates like "electron" behave like mass terms in cases for which there is uncontroversially a truth value. In other words, for those assertions about quantum systems that are uncontroversially true or false, can we consistently interpret talk of quantum particles (the things that are called "anonymeres") as talk involving the sort of continuous substances taken to be the referents of canonical mass terms? From the metaphysical perspective, there is the question of what specific content we

[18]How could this maneuver fail? How could QM force (or at least encourage) us to reject the semantics of mass predication and embrace references to particles as canonical count terms? If the theory entailed that, in certain circumstances one could say there are exactly n particles and, furthermore, one could determine which is which – by, for instance, a procedure for re-identifying particles – this would make a mass predication reading difficult. That is, if the theory implied that the things in question behave like classical individuals, this would be a problem for the mass term view. But the theory doesn't say this. It is the very fact that QM seems to deny this possibility that motivates the attempt to introduce anonymeres in the first place.

should assign to quantum terms as mass terms in order to understand the truth values of quantum propositions. To what, exactly, do they refer? What must the world be like if such claims are true? What could such continuous substance be? To approach these metaphysical questions, we must already have fixed a semantics, a way in which meaning is attached to terms. In the remainder of this section, I consider the semantic issues, leaving worries about the attendant metaphysics for the next section.

Do the truth conditions of quantum claims conform to the conditions (MT1) – (MT5)? In the context of non-relativistic quantum mechanics, the only properties one can uncontroversially ascribe to a quantum system correspond to symmetric operators (operators which commute with the so-called permutation operators).[19] These are properties that do not reference any specific particle. They allow you to make claims such as:

(S1) One electron in the system is spin-up, the other spin-down.

(S2) The total angular momentum is J.

It is also possible, without violating the postulates of QM to make assertions like:

(S3) The particles in the box are pions.

(S4) Lithium atoms have 3 electrons.

These claims are clearly about something, and given the discussion in the first half of this essay, we can conclude that whatever that something is, it cannot be counted.[20] In other words, it must be 'stuff', in the philosophical parlance. So what of the remaining conditions? Let's start with division. French and Krause (2006) insist that terms like

[19] For an overview of the relevant formalism, see (Messiah and Greenberg 1964) and (Hartle and Taylor 1969).

[20] Note the distinction between counting water and counting particular volumes, samples, or regions containing water.

"proton" cannot be read as mass nouns since they fail to divide their reference. A limited division of reference is necessary for mass predication, and so it seems quantum sortals cannot be cast in the mold of mass term predications. But notice that when French and Krause provide explicit examples of the criteria of identity for the ostensible sortals, they speak of *systems*. They say, for instance, that "physicists have the possibility of recognizing...whether a given physical system is, say, an electron system or not" (French and Krause 2006, p350). "Electron" in this case is predicated of a system; it is not acting as a sortal. This is exactly analogous to the case of water – one can count bodies of water (systems of water) using criteria of identity that pick out particular spatial regions (e.g., a lake or a bucket). But "water" is predicable of the stuff in each region. The term water in these cases still divides its reference. The stuff in half of a lake is still water. Likewise for physical systems individuated by picking out a particular region of space or spacetime. "Electron" clearly does divide its reference this way, since electron systems have spatial parts that themselves contain electron (in that the properties of mass, charge, and spin may manifest there). Half of the volume we may identify as a quantum dot may still be said to contain a stuff we call "electron" (or perhaps more aptly, "electron-stuff").[21] Each half can be attributed all of the same kinds of properties as the whole. For instance, each has an expected charge current that varies with applied voltage. To be clear, I'm translating assertions such as "There is a 30% chance of finding the electron in region X," as "In region X, there is a 30% chance of finding 1 unit of electron-stuff." If electron-stuff in this sense can be predicated of a region A and P is a spatial part of A, then QM guarantees that electron-stuff can be predicated of P, thus dividing reference.

Not to belabor the point, but it's important to distinguish between the mereology

[21] I am assuming that electron-stuff may be said to be present in any region of space where the wavefunction has support. The entire region in which the square of the wavefunction is non-zero is special in that one can say with certainty that it contains one unit of electron-stuff.

of predication as reflected in the distributivity and cumulativity conditions, and the laws governing the combination of quantities. It's true that if we insist on trying to read "electron" as a canonical count term, then its referent must be an object with a particular mass, charge, spin, etc. Half of such a thing would not itself be an electron. However, framing things this way would be to insist on a metaphysical package that exceeds the set of uncontroversial empirical propositions sanctioned by the theory – it's more than we need in order to capture the truth conditions for empirical statements of quantum theory. It is true that quantities of mass, spin, and charge are tied together. But all the theory forces on us is an association between measurements made on the same spatial region – if a mass of m_e is measured, then it's possible to have simultaneously measured a charge of e. We can still rephrase claims about the system in terms of "electron" as claims about a stuff, and whatever we call that stuff, it divides its reference. But the way in which properties associated with or characteristic of electron-stuff combine is more complex. Some properties – like mass and charge and spin – are bound in fixed, invariant proportions, others are not. This particular feature of quantum properties is not so strange in that much the same is also true of the referents of ordinary canonical mass terms like "water." Just as every sample of electron-stuff has a fixed, characteristic mass-charge ratio, so every sample of water has a fixed specific heat. That is, the association between heat capacity and mass is the same for all bits of water. Division of reference thus does not stand in the way of treating so-called quantum sortals as misconstrued mass terms. A similar argument can be given for cumulativity.

What about measurability? Can we say how much electron there is? I concede that references to quantum particles have one of the features French and Krause attribute to so-called "quantum sortals": "Instead of a criterion of identity, there is a *number criterion*, a principle which enables us to say that in certain situations [a given] predicate truly applies to a certain number (generally finite) of entities, yet sometimes there is no counting process associated with them"(French and Krause 2006, p350). But this feature is easily rephrased to show it fits the criteria of mass term semantics equally well: "Instead of a criterion of identity, there is a measure which enables us to say that a given predicate truly applies to a particular quantity (generally finite) of stuff. There is generally no counting process associated with the measure." This rephrased condition is a generic feature of mass term predication, in particular it is just condition (MT5) above.

To put the matter more generally, I claim that, like "water", "electron" or "electron-stuff" is properly attributed quantity, not number. There is, however, an important difference with respect to substances (if there are any) that satisfy a strict version of Condition (MT2). Suppose, for example, that space is infinitely divisible. Then the possible quantities in some unit of measure that can be used to quantify volumes of space are continuous, or, more carefully, they are representable by a continuous set of values. Quantum stuff, on the other hand, is notoriously discrete (hence the name). That is, in some given unit of measure, electron-stuff – if 'stuff' it be – comes in quantities representable by a discrete set of values. But this is also true of many ordinary mass terms that satisfy only the weak version of Condition (MT2). Here it may help to think about sand. In English, 'sand' acts as a canonical mass term. It divides reference, it is always quantified with a unit of measure, and we usually act as though the values we can obtain (for either volume or mass) are continuous. But, of course, they are not. For any given sand pile or for the aggregate of all sand, there are only finitely many values of mass or volume, because sand always comes in discrete grains. The point is that a term can satisfy the measurability condition and function as a mass term and yet be quantified via units of measure that take on only discrete values. I don't mean this as a metaphysical thesis, but a semantic one. We know what we *mean* by sand, and the fact that it quantifies as it does does not change the fact that it functions as a mass term vis-a-vis reference. So too, I claim, for electron-stuff.

By this point, it should be clear that there is a close parallel between sentences like (S1)-(S4) and the following:

(S5) Half of the liquid is vinegar, the other half is water.

(S6) The total mass is m.

(S7) The liquid in the beaker is glycerol.

(S8) A shot of whisky has 0.6 oz. of alcohol.

In each of the sentences (S1)-(S4), we see particle terms being used nominatively or predicated of a system just as the ordinary mass terms in (S5)-(S8) are used nominatively or predicated of individual things. We get the right semantic behavior if, for instance, we rephrase (S1) to read: "Half of the electron-stuff is spin-up and half is spin-down." This has precisely the same structure as (S5) where "electron-stuff" acts like the mass term "liquid". In similar fashion, we can rephrase (S4) as, "A standard unit of lithium atom has 3 units of electron-stuff." This is awkward but clearly parallel in form to (S8). My rephrased sentences look strange, but that is because the weight of history is behind using words like particle and electron syntactically as count terms. Insofar as sentences involving those terms are uncontroversially meaningful, however, they act like disguised mass terms. Only metaphysical prejudice could keep us from taking their semantic role at face value.

Confusion was perhaps inevitable in that physicists and philosophers treat "electron" syntactically like a count term but semantically like a mass term. Perhaps it would be less confusing to substitute "electron-stuff" or "unit of electron-stuff" for "electron", depending on the circumstance (as I've often done above). "Electron" is frequently predicated of systems, as in "the mobile charge carriers in the quantum dot are electrons", but it is also used nominatively when quantified, as in "atoms with three electrons." In the former case, "electron-stuff" is a more apt substitution, as in "The mobile charge-carrying material in the quantum dot is electron-stuff". In the latter case "unit of electron-stuff" may be more appropriate, as in "atoms bearing three units of electron-stuff." The point is that the term "electron" and other quantum terms like it meaningfully function only as mass terms – terms whose referents satisfy (MT1) - (MT5). I suggest that the notion of an anonymere has arisen from a confusion between number and quantity, count term and mass term. Once we see that we are dealing with mass term predication, there is no need to invoke anonymerity.

Perhaps this seems too easy. Perhaps one is inclined at this point to raise one or another famously odd aspect of the quantum theory as a stumbling block. Surely, the thought goes, quantum mechanics says things about the world that are dramatically different from what canonical mass term semantics suggests about ordinary samples of stuff like water, and so ordinary mass predication cannot possibly capture the truth conditions of quantum mechanics. There is some truth to this objection – the behavior of physical systems according to quantum mechanics involves profoundly non-classical elements. However, these non-classical features generally concern relations amongst observables or properties, and do not stand in the way of a canonical mass term rehashing of informal descriptions of quantum systems.

To isolate one central example, let's consider entanglement. On the view I've been defending, what are we to say of entanglement? Syntactically at least, it seems natural to say that particles are entangled. If we translate such a claim as "particle-stuff is entangled", the result is hard to parse. But this is the wrong translation. Entanglement is in the first place a feature of quantum states. There are a variety of ways to spell out precisely what counts as an entangled state,[22] but all agree that entangled states involve

[22]E.g., a pure state which, when expressed as a density operator, yields only reduced density operators that are not pure states.

correlations between measurement outcomes that are not possible for classical systems, with or without hidden variables. Note that I said "measurement outcomes" and not particles. If we view particles as individuals of some sort, then we could sensibly talk about entanglement in terms of correlations amongst (measurable) properties of distinct particles. That's the old, unworkable count term view. In the alternate semantics I'm urging, it would not be correct to say that the electron-stuff is entangled, but rather that states of particular regions or bits of electron-stuff are entangled. That is, entanglement is a feature of the probability distribution over properties of distinct portions of electron stuff (e.g., electron-stuff occupying non-overlapping spatial regions). That doesn't mean entanglement isn't noteworthy or strange. The point is rather that claims about entanglement behave semantically like claims involving mass term predication, albeit with weird relations or laws connecting properties of the stuff so predicated.

Though I obviously lack the space here to elaborate a full semantics and logic of mass terms in quantum mechanics, the lesson is clear. Insofar as talk about ostensible anonymeres is meaningful, it exhibits all the semantic (if not syntactic) features of talk about mass terms. If we take this to be the case, then the necessary semantics is no more (or less) mysterious than the semantics of mass-terms. But then the notion of anonymere is superfluous. Here, then, is the semantical dilemma. On the one horn, we can attempt to interpret talk about anonymeres directly along the lines of quasi-set theory. But we lack a suitable semantics for doing so. On the other horn of the dilemma, we can embrace a semantics of mass terms. But then we are not talking about anonymeres, we're talking about stuff with properties expressed via the semantics of canonical mass terms.

5.1 Metaphysics

But what is electron-stuff? What is it that is measured in discrete quantities? These are metaphysical questions, not semantic ones, and I've been advancing a semantic thesis. Nonetheless, something must be said about the possibility of answering such questions. If in fact there is no plausible metaphysical theory – no interpretation *given a standard semantics* – that fits my reading of quantum terms as mass terms, then we would indeed be forced to consider more radical options like scrapping our standard semantics to make room for something like entities without identity. But this is simply not the case. There are perfectly good interpretations that are compatible with the mass term reading given a traditional semantics.

In quantum chemistry, for instance, an approach called 'Atoms in Molecules' or AIM is an important competitor to Density Functional Theory (Bader 1990; Bader 1991; Popelier 2000). Broadly speaking, AIM treats the electron density function ρ (the square modulus of the many-electron wave-function) as the primary theoretical entity from which all chemically relevant properties can be derived. More specifically, atoms, molecules, and chemical bonds can all be defined in terms of geometric features of the gradient field on ρ. Atoms, for instance, consist of the union of an attractor (a point at which field lines converge) and its basin (the region of space from which the convergent field lines originate). In other words, the electrons and the atoms and molecules they partly compose are treated as distributed quantities of stuff occupying a spatial region. To put it differently, one of the fundamental objects in the ontology of AIM is a field of electron density that fills space the way that an ideal fluid does. Of course, AIM continues to treat atomic nuclei as discrete objects, but this is largely for practical computational reasons. There is no reason in principle the density-functional approach couldn't be extended to all particles.

Philosophers have produced similarly concordant interpretations of quantum mechanics. Lavine (1991, p266-268), for instance, urges a view coarsely sketched in five parts (his numbering):

1') Objects are represented as being made of simple kinds of stuff. The simple kinds of stuff have smallest possible amounts.

2') The properties, relations, and behavior of complexes are connected with those of simple kinds of stuff of which they are composed.

3') A theory of the properties, relations, and behavior of the smallest amounts is given.

$3\frac{1}{2}$') A theory of how the smallest amounts combine to make larger quantities is given.

4') The properties, relations, and behavior of the complex system are explained or predicted by applying the theory of the properties, relations, and behavior of the smallest amounts of the simple kinds of stuff and the theory of how these amounts combine to infer the properties, relations, and behavior of the particular configuration of kinds of stuff that occurs in the system.

He even explicitly connects this metaphysical view with semantics, noting that "[a]bandoning talk of indiscernible individuals in favor of talk of stuff eliminates awkward metaphysical problems at a comparatively cheap price: we admit some irreducible mass terms into our discourse about the physical world" (Lavine 1991, p266).

A much more detailed interpretation along these lines was developed simultaneously by Wallace and Timpson (2010) and myself (Jantzen 2010). These remarkably similar proposals represent a more radical departure from traditional views. In the case of ordinary mass terms like gold or water, we think of ourselves as referring to a quantity[23] of material substance with certain properties. In AIM, "electron" refers to a charge/matter field in much the same way. In the accounts of Wallace and Timpson and myself, however,

[23]See (Cartwright 1970) for a cogent discussion of the notion of "quantity" as I use it here.

the substance in question is spacetime or space, respectively. In my interpretation of non-relativistic quantum mechanics, regions of space are conventional individuals and are the basic ontological elements. So, for instance, 'electron' applies to any region of space with a particular suite of properties, namely a non-vanishing probability of exhibiting mass and charge in characteristic increments and a particular fixed proportion. More generally (in the sense of serving as an appropriate interpretation for relativistically correct quantum field theories), Wallace and Timpson (2010, p712) "associate a set of properties (represented by a density operator) to each region of spacetime." In both cases, "electron" does not refer to a sort of thing which can be counted, but rather to a kind of stuff, a kind of spatial (or spatiotemproal) stuff. In other words, it refers to the sort of thing that respects the properties of the canonical mass term referent. Note, incidentally, that we could give a similar reading to the referents of ordinary mass terms in our folk ontology. Water, for instance, can be seen as space with certain measurable properties namely mass (or density), transparency, a certain taste (or lack thereof), etc.[24] Mass term semantics works equally well whether we are talking about undifferentiated material substance or spacetime. Of course, the latter sounds a bit weird. But whether it ultimately makes sense to treat regions of spacetime as fundamental objects in our ontology is beside the point. What matters is that such a metaphysical view fills in plausible referents for quantum particle talk such that truth values come out right given the standard semantics of mass terms. In a loose sense, the detailed metaphysical theories of Wallace and Timpson and myself are plausible models of the quantum theory.

To be clear, I do not mean to endorse any of these theories as correct (though one may be). I intend only to use them as a proof by example that the relevant impossibility claim is false – metaphysics does *not* stand in the way of a mass noun reading. Nor am I claiming that any such interpretation will suddenly lift the veil of mystery that

[24]Bennett (1984) argues that Spinoza had such a metaphysic in mind. That is, he saw ordinary objects as modes of regions of space. I'm grateful to Walter Ott for pointing out this connection to me.

hangs over quantum mechanics. Quantum phenomena are still bizarre relative to the happenings of everyday experience. My point is that, if possible, it is better to embrace a strange metaphysics than a strange metaphysics *and* a radical departure with the ways in which we attach meanings to words.

6 Conclusion

I have argued that the definition of anonymeres as entities that differ *solo numero* is incoherent if taken literally, and that there appears to be little hope of providing an alternate account of anonymerity with the desired characteristics. This is in part because of close conceptual ties between identity and cardinality, and in part because the very notions of reference and quantification in traditional semantics presuppose individuality. To make sense of anonymerity would require relinquishing not just traditional notions of cardinality, but our most robust theories of meaning as well.

I also claimed that the sort of assertions in physics that inspired the introduction of anonymere talk are meaningful, but only when we recognize that words long used as count terms are functioning as mass terms. Fortunately, reinterpreting quantum terms as mass terms requires a far more modest logical and semantic project than producing a theory of anonymeres. In fact, interpretations of quantum mechanics that treat references to particles in this way can already be found in the philosophical and scientific literature. Adopting a mass-term semantics for quantum particle talk is thus perfectly compatible with drafting serious and intelligible interpretations of quantum physics.

Whatever full interpretation is ultimately adopted, particle terms are mass terms. In this sense, they are not so exotic. There is no need to pursue a radically revisionist semantics just to make room for anonymeres. The breathless story of quantum mechanics forcing us into a brave new world of metaphysical possibility in which the very notion of self-identity fails to apply is surely full of sound and fury. I have argued that key terms in this story signify nothing.

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