Why talk about 'non-individuals' is meaningless

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Abstract

It has been suggested that puzzles in the interpretation of quantum mechanics motivate consideration of 'non-individuals', entities that are numerically distinct but do not stand in a relation of identity with themselves or non-identity with others. I argue that talk about non-individuals is either meaningless or not about non-individuals. It is meaningless insofar as we attempt to take the foregoing characterization literally. It is meaningful, however, if talk about non-individuals is taken as elliptical for either nominal or predicative use of a special class of mass-terms.

It is something of a truism that we ought not to read our metaphysics off of the structure of our language. But it is also the case that any metaphysics we might know and describe must be known and described through the medium of language. The way in which meanings attach to words must therefore circumscribe, however loosely, the possible metaphysical theories we are capable of articulating. There are bounds to what can be meaningfully asserted. To stray outside these is to utter nonsense.

This is precisely what has happened in a trendy corner of the philosophy of physics. Vexed by puzzles in the interpretation of quantum mechanics, a number of philosophers have begun to talk about "non-individuals." This, I suggest, is unhelpful. Either talk about non-individuals is nonsense, or it is not *about* non-individuals.

1 What's a non-individual, and why would anybody talk about one?

The impetus for talk about non-individuals is quantum mechanics. The reasons are well-documented elsewhere (French and Krause, 2010a), but to give a sense for how such arguments go, I'll sketch a standard trope in the literature that draws upon features of quantum statistical mechanics.[1] Ignoring some technical details of quantum state representations, the argument runs like this. Take it as a premise that all distinct (if not qualitatively distinguishable) configurations of particles and properties are equally probable. We might say that each distinct state receives an equal portion of probability. Consider, for example, the case in which we have just two particles, call them p_1 and

[1] See, e.g., (Post, 1963; Reichenbach and Reichenbach, 1999; French, 2000; French and Krause, 2010a).

p_2, and two distinct bundles of properties that may be predicated of each, call them M_1 and M_2. If we denote the proposition that p_i possesses properties M_j by $M_j(p_i)$, then classically there are four distinct states in which we might find such a system:

1. $M_1(p_1) \wedge M_1(p_2)$,
2. $M_1(p_1) \wedge M_2(p_2)$,
3. $M_2(p_1) \wedge M_1(p_2)$, and
4. $M_2(p_1) \wedge M_2(p_2)$.

The probability of finding one particle in M_1 and one in M_2, irrespective of which is which, is given a portion of probability twice as large as the other states (1/2 versus 1/4). This simply reflects the fact that two distinct states match that description. According to QM, however, states in which one particle has properties M_1 and the other has M_2 must be granted a portion of probability as though there is but a single state.[2] In other words, the two possibilities (2) and (3) are treated as one. Thus, we have the elements of a modus ponens. If (2) and (3) were distinct states of the world, then each state would have to be assigned a unit weight relative to the other possibilities. Quantum statistical mechanics tells us that in fact the two states *combined* receive a unit of probability. Therefore, quantum state representations with particle names permuted do *not* represent distinct states. This is supposed to suggest that in the case of quantum particles, "...labels are otiose" (French, 1998, p95).

There are many difficulties with the argument sketched above, and I haven't the space here to give the technical issues their due. However, let us take the argument at face value and consider the notion of 'non-individual' it suggests. According to the principal proponents of this view, *non-individuals* are entities that are numerically distinct and yet fail to stand in relations of self-identity. In other words, there is no relation that holds only between a non-individual and itself. As French is fond of putting it, they differ *solo numero* (French and Krause, 2010a). If quantum particles are non-individuals, this would explain their strange statistics. For there to be a difference between $M_1(p_1)$ and $M_1(p_2)$, it must be the case that the subscripts label distinct particles. But if that were true, the particles would stand in a relation that acts like self-identity. Specifically, particle 1 is that which uniquely bears the label p_1. Since non-individuals cannot stand in such relations, it must be that they cannot be uniquely referenced. In that case, one cannot assert a distinction between $M_1(p_1)$ and $M_1(p_2)$. Nonetheless, there are determinately many of them in a collection, just as there are determinately many hydrogen atoms in a given flask. The new category of non-individual handily accounts for these facts.

[2]Roughly speaking, there are actually two possibilities corresponding to two basic kinds of particle. For bosons, the states in which one particle possesses M_1 and the other M_2 would receive a combined probability of 1/3 as would each of the states $M_1(p_1) \wedge M_1(p_2)$ and $M_2(p_1) \wedge M_2(p_2)$. For fermions, the only possible state is that in which one particle possesses M_1 and the other M_2.

But what can it mean to differ *solo numero*? It's easier to say what this cannot mean. If 'numerical distinctness' is understood in anything like the usual way, then non-individuals cannot be numerically distinct and yet fail to be identifiable. In the standard set-theoretic definition, cardinality essentially involves a notion of mapping or correspondence that is conceptually equivalent to labeling. Specifically, the cardinal associated with a set A (intuitively, the number of things in A) is the smallest ordinal number n such that there is a bijection from the elements of A to the elements of n. To put it more plainly, two sets are the same size as one another if their elements can be brought into one-to-one correspondence, that is, if the elements of one can be used to uniquely label elements of the other. Ordinals are just specially constructed sets whose elements are rigidly ordered (i.e., there is a binary relation, $<$, such that for every pair of elements a and b, either $a < b$ or $b < a$). The ordinal number 3, for instance, is the set $\{0, 1, 2\}$. The cardinality of a set is the smallest ordinal of the same size. Put yet another way, the cardinality of a collection is what we get by a generalized counting procedure in the intuitive sense of counting. After all, counting is really just a sort of indexing by which we point at distinct things and label them by saying "one", "two", "three", etc.

The relation between identity and cardinality is not a metaphysical fact, but rather a semantic one. I do not mean to suggest that there is some metaphysically necessary association between identity and cardinality. Rather, I am claiming that what it *means* for entities in a collection to be numerically distinct is for the collection to possess a cardinality greater than one. And part of what it *means* for a collection to possess a definite cardinality — on any standard account of cardinality — is for the entities to be identical with themselves and no others in the collection.[3]

It is important to note that I needn't stake a claim about the nature of the identity relation. Cardinality requires an identity relation only relative to the collection being counted. But this identity may be grounded in multiple ways. The identity relation may be grounded internally by predicates belonging to the same structure as the collection being considered. For example, it is at least contingently true that no two leaves in my garden have exactly the same shape. Shape serves to distinguish them, and this distinguishability underwrites an identity relation good enough to assign a cardinality to the set of leaves. Alternatively, the identity relation for a collection may be grounded externally with no discernible differences in the elements of the collection within the structure in which it is presented. Consider, for instance, the vertices of a perfectly symmetric undirected graph. Every vertex stands in precisely the same relations as every other — there are no graph properties or relations to distinguish them. But we could always label the vertices.

This point is worth belaboring as it generally overlooked in the literature. To give one example, Domenech and Holik (2007) offer what they take to be a definition of cardinality worthy of the name and suitable for the case of a collection of non-individuals. Suppose we have such a collection, X. Informally speaking, their proposal depends upon

[3] See (Jantzen, 2011) for a more thorough argument along these lines, and (Arenhart, 2012) for a rebuttal.

the notion of 'quasi-singleton' defined with respect to X. For some $x \in X$, the quasi-singleton $\langle x \rangle$ is a collection whose only sub-collections are the empty collection or itself.[4] This is certainly a feature one would expect for a collection that contains just one thing. The cardinality of X is then defined by constructing a series of sets, each of which is derived from the previous by removal of quasi-singleton. The series terminates (if at all) when there is nothing left to remove. Essentially, the cardinality of X is given by the length of the chain of derived sets. This procedure is ingenious and perfectly consistent. However, it fails at its stated aim. As Jantzen (2011) proves, the notion of a quasi-singleton as Domenech and Holik define it provides an identity relation on the elements of X *relative to the structure in which it is presented.* In other words, relative to the collection X and all collections that may be constructed from X, membership in a particular quasi-singleton acts as an identity relation. It is no surprise then that we can sensibly count using this procedure. Arenhart (2012) has attempted to rebuff Jantzen's critique by pointing out, quite rightly, that the putative identity relation is not a first-order identity relation on the whole universe of non-individuals and collections of them, supposing there are others in it besides those of X. My point is simply that this is irrelevant. The relation in question is acts like an identity relation for the collection one is counting. What's supposed to make non-individuals special is that a collection of them possesses a distinct cardinality without any relation relative to that collection serving to distinguish them one from another. We wouldn't worry whether electrons in a collection are in some global sense *really* non-individuals if it were always the case that in any given collection of electrons, there is a property that distinguishes them from each other. Then we could simply count property bundles, as it were, in the old-fashioned way of counting. There would be nothing to motivate talk about 'non-individuality'.

When I claim that the meaning of 'cardinality' is dependent upon the meaning of 'identity', I intend only to invoke a bare concept of sameness and difference. Perhaps identity always requires distinguishability as Leibniz would have it.[5] Perhaps not. Perhaps identity is primitive in some way, independent of all other intrinsic properties as Adams (1979) would have it. Perhaps not. The point is that some relation with the features of identity must obtain for a collection if there is to be a definite number of things in the collection. If you think there can be a definite number of things all of which are qualitatively indistinguishable with respect to all of the predicates of the language in which cardinality is expressed, then that identity relation must be either primitive or grounded in some features not expressible in the language. But that is not a decision we need to make. It suffices to note that the assertion that there are n things in a collection is to assert that there is a relation of identity amongst those things such that each is identical to itself and not any of the others in that collection. To say that the members of a collection are non-individuals, is to deny even a weak, contextual notion of identity. Therefore, the notions of 'cardinality' and 'numerical distinctness' must mean something

[4]The proposal of (Domenech and Holik, 2007) is spelled out in terms of quasi-set theory discussed below. Thus, the technical definitions involve quasi-sets. However, to efficiently convey the gist of the proposal, I am using the neutral term "collection" instead.

[5]See, e.g., (Saunders, 2003).

else when invoked to define or describe non-individuals. So what is meant?

2 Quasi-sets as quasi-solutions

When intensional definitions that employ established terms in the definiens prove inadequate to convey the sense of a radically new concept, one might instead attempt to establish the precise meaning of a new term by providing an account of the semantic role of that term. A particularly effective tool for doing so is formal axiomatization. Axioms in a formal language are satisfied by a subset of all possible models. Examining those models that satisfy the axioms tells you something about what features of the world (or possible worlds) correspond to an unknown term. In this way, concepts can be defined, or at least circumscribed, by formal axiomatizations. So, for instance, if you wish to understand geometric concepts like 'triangle', you could consider Euclid's axioms. If you want to understand the general notion of distance, you could consider the axioms satisfied by all metrics, and so on. A prominent attempt to do this for non-individuals comes from Decio Krause,[6] who axiomatized a theory of "quasi-sets"(Krause, 1992).

The formal theory of quasi-sets, $\mathfrak{Q}$, is presented in a first-order language. It was designed as a conservative extension of ZFU, the Zermelo-Frankel axiomatic theory of sets with *urelemente*. In other words, there is a 'copy' of ZFU within $\mathfrak{Q}$ such that theorems of ZFU are theorems of $\mathfrak{Q}$ as well. What's different about $\mathfrak{Q}$ is the introduction of a second kind of *urelemente* or "atom". In classical set-theory, the atoms are presumed to stand in relations of identity such that each is identical with itself and no other. Krause and French call these "M-atoms." In $\mathfrak{Q}$, these classical M-atoms are complemented by others which can be members of so-called 'quasi-sets', can stand in relations of indistinguishability (e.g., each is indistinguishable from itself), but do not stand in relations of identity. They call these "m-atoms." If x and y are m-atoms, then "$x = y$" is not a well-formed expression of $\mathfrak{Q}$. This reflects the fact that, in the intended interpretation, there is supposed to be no fact of the matter whether one m-atom is identical with another. Of course, the point of the formal theory is to pin down this intended interpretation. The m-atoms are the formal counterparts of non-individuals. By interpreting m-atom terms in $\mathfrak{Q}$, we're supposed to get a grip on what talk about non-individuals refers to. It is in attempting to interpret $\mathfrak{Q}$ however, that we encounter a problem.

The problem is that formal theories in first-order languages like ZFU are standardly interpreted according to Tarskian semantics. Speaking coarsely, an interpretation is a structure that includes the specification of a domain of discourse, D, as well as properties, relations, and functions on that domain. The domain of discourse is understood to be a set of objects (whether mathematical or physical objects). Properties and relations amongst the objects in the domain are defined extensionally, i.e, each binary relation R is presented in the interpretation as a set of ordered pairs $\langle x, y\rangle$ such that x stands in R to y. Sentences in the formal language are interpreted by mapping names and variables to objects in the domain and predicate symbols to properties and relations on

[6]The theory was later revised in collaboration with Steven French (French and Krause, 2010b,a).

D. Sentences in the language are true in a structure (i.e., a particular domain and set of extensionally defined predicates) just if the interpreted sentence is true. If all sentences of the theory are true, the structure is said to be a model of the formal theory.

The theory $\mathfrak{Q}$ does have models in the Tarskian sense. In fact, it can be modeled by the sets of ZF.[7] That's how French and Krause (2010a) prove the consistency of their axioms. The problem is that the objects in the domain of discourse must have identity. In part, this is because that is how the elements of classical sets are conceived. It was on these grounds that da Costa et al. (1995) expressed concern about finding an appropriate semantics for $\mathfrak{Q}$. But in part the problem is implicit in the general form of Tarskian semantics, whether or not we consider the domain a "set." I said that names and variables "correspond" to or "map" to objects in the domain of discourse. For this to be the case, it must make sense to assert that a particular constant or variable refers — and refers uniquely — to an object. The m-atoms of $\mathfrak{Q}$, or more accurately, the things to which variables in the theory are supposed to correspond lack this feature. If $\mathfrak{Q}$ is to be interpreted in terms of non-individuals, there cannot be a mapping or reference relation between an m-atom symbol or term in the theory and a unique entity. For this reason, the interpretation of $\mathfrak{Q}$ requires a new semantics.

I do not mean merely that it requires us to consider structures other than classical sets as models. The problem is rather deeper. The relation between formal sentences and possible worlds in the Tarskian scheme is one in which symbols label or uniquely and unequivocally denote particular objects. Furthermore, we're supposed to be able to describe properties and relations extensionally with sets of ordered tuples. Neither of these is possible for non-individuals. What we need is a whole new semantics for the formal theory.

In an attempt to find an alternative to Tarskian semantics, Arenhart and Krause(2009) have undertaken the exercise of constructing both the logic underlying $\mathfrak{Q}$ and a formal semantics for the theory in terms of $\mathfrak{Q}$ itself. That is, they have used the theory of quasi-sets to define the language and state the axioms of $\mathfrak{Q}$, and to provide a semantics for interpreting sentences of the theory. As they point out, something similar can be done for classical ZF. Depending on one's purpose, there is nothing illicit about using one and the same language as both the object language and the metalanguage. But if the purpose is to use axioms along with a given formal semantics to understand a new concept, it is problematic to describe the semantics in terms of the very concept we are trying to understand. The question is whether we can use the new notion of a 'model' of the axioms of $\mathfrak{Q}$ to help us understand terms like 'non-individual'.

To see why there might be a problem with this strategy, consider the classical case. Tarskian semantics shares only a handful of notions with set-theory, namely the idea of a set, ordered tuple, and mapping or assignment.[8] These notions all have relatively clear, intuitive meanings independent of any particular formal theory or mathematical exposition. Thus, if one wanted to understand an idea like set membership or power set,

[7]That is, by Zermelo-Frankel set theory without *urelemente*, only sets.

[8]The fact that Tarskian semantics does not presume the full conceptual resources of a single theory of sets is demonstrated by the existence of 'non-standard' models of set-theory.

then one could use Tarskian semantics to investigate models of a set of axioms without circularity or contradiction. But if one wanted to understand, say, identity there would be trouble. Identity is one of the notions essential to Tarskian semantics, and so looking at models of a theory of identity would teach us nothing about identity. We would have to know what identity means before we could understand what models are in the first place. For understanding non-individuality, quasi-set theory is similarly impotent. We cannot understand non-individuality using a semantics stated in terms of $\mathfrak{Q}$ because the relevant notions for stating the semantics derive from the notion of non-individuality. The portions of quasi-set theory we need in order to understand the proposed semantics involve precisely the notion we are trying to learn about by employing a formal semantics. Just as classical formal logic with Tarskian semantics is not very helpful for teaching us what identity is, quasi-set theory with its mysterious semantics is useless for gaining an understanding of non-individuals. For this purpose, the relation between formal semantics and meta-language is viciously circular.

3 A semantical dilemma

If we cannot understand non-individuals through the formal axiomatic theory of quasi-sets, how should we understand talk about non-individuals? I suggest we look again to physics. It was physics that motivated such talk in the first place, specifically puzzles about quantum particles. It is clear that at least some talk about quantum particles is meaningful. Physicists successfully coordinate expectations, agree on logical relations, and by and large agree on truth conditions for claims about systems of quantum particles. If we assume they are successfully talking about 'non-individuals', what can be said about the meaning of this term? The answer is surprisingly mundane. Those assertions about particles or systems of particles which undeniably have semantic content behave not like count terms as one would expect for either classical particles or non-individuals, but rather like mass terms.[9]

In the context of non-relativistic quantum mechanics, the only properties one can uncontroversially ascribe to a quantum system correspond to symmetric operators (operators which commute with the so-called permutation operators).[10] These are properties that do not reference any specific particle. They allow you to make claims such as:

(S1) One electron in the system is spin-up, the other spin-down.

(S2) The total angular momentum is J.

It is also possible, without violating the postulates of QM to make assertions like:

(S3) The particles in the box are pions.

[9] I agree with (Koslicki, 1999) that the mass/count distinction properly applies to occurrences of a term, not the term itself. For ease of exposition, however, I will elide this distinction.

[10] For an overview of the relevant formalism, see (Messiah and Greenberg, 1964) and (Hartle and Taylor, 1969).

(S4) Lithium atoms have 3 electrons.

Importantly, there is a close parallel between sentences like (S1)-(S4) and the following:

(S5) Half of the liquid is vinegar, the other half is water.

(S6) The total mass is m.

(S7) The liquid in the beaker is glycerol.

(S8) A shot of whisky has 0.6 oz. of alcohol.

In each of the sentences (S1)-(S4), we see particle types being predicated of a system just as the ordinary mass-nouns in (S5)-(S8) are predicated of individuals. We might rephrase (S1) to read: "Half of the electron-stuff is spin-up and half is spin-down." This has precisely the same structure as (S5) where "electron-stuff" acts like the mass-noun "liquid". In similar fashion, we can rephrase (S4) as, "A standard unit of lithium atom has 3 units of electron." This is awkward but clearly identical in form to (S8). My rephrased sentences look strange, but that is because the weight of history is behind using words like particle and electron syntactically as count-nouns. Insofar as sentences involving those terms are uncontroversially meaningful, however, they act like disguised mass-nouns. Only metaphysical prejudice could keep us from taking their semantic role at face value.

One might at this point insist, as French and Krause (2010a) do, that terms like "proton" cannot be read as mass nouns since they fail to divide their reference. A limited division of reference is necessary for mass predication, and so it seems quantum sortals cannot be cast in the mold of mass-noun predications. But this I suggest is another instance of the the mistake diagnosed above. When French and Krause provide explicit examples of the criterion of identity, they speak of *systems*. They say, for instance, that "physicists have the possibility of recognizing...whether a given physical system is, say, an electron system or not" (French and Krause, 2010a, p350). "Electron" in this case is predicated of a system; it is not acting as a sortal. It clearly does divide its reference, since systems do have parts. That they have minimal parts — that some parts of an electron system are not themselves electron systems — is not a worry; the same is true of many ordinary mass-nouns like "furniture." There is thus nothing in the way of treating particle terms as mass-nouns.

Confusion was perhaps inevitable in that physicists and philosophers treat "electron" syntactically like a sortal but semantically like a mass-noun. Perhaps it would be less confusing to substitute "electron-stuff" or "unit of electron" for "electron", depending on the circumstance. It is often predicated of systems, as in "the charged particles in the quantum dot are electrons", but it is also used nominatively when quantified, as in "atoms with three electrons." In the former case, "electron-stuff" is a more apt substitution, as in "The charged material in the quantum dot is electron-stuff". In the latter case "unit of electron" may be more appropriate, as in "atoms bearing three units of electron." The point is that the term "electron" and other quantum terms like it meaningfully function only as mass-nouns. In other words, the semantic roles of particle talk in quantum physics just are the semantic roles of mass-nouns. I suggest that the notion of a non-individual has arisen from a confusion between number and quantity, count-noun and mass-noun. Once we see that we are dealing with mass-noun predication, there is no need to invoke non-individuality.

Though I obviously lack the space here to elaborate a full semantics and logic of mass-terms in quantum mechanics, the lesson is clear. Insofar as talk about ostensible non-individuals is meaningfully, it exhibits all the features of talk about mass-terms. If we take this to be the case, then the necessary semantics is no more (or less) mysterious than the semantics of mass-terms. But then the notion of 'non-individual' is superfluous. Here, then, is the semantical dilemma. On the one horn, we can attempt to interpret talk about non-individuals directly along the lines of quasi-set theory. But we lack a suitable semantics for doing so. On the other horn of the dilemma, we can embrace a semantics of mass-terms. But then we are not talking about non-individuals, we're talking about individual somethings with properties expressed via the mass-predication of a term.

4 Conclusion

I have argued that the definition of non-individuals as entities that differ *solo numero* is incoherent if taken literally, and that there appears to be little hope of providing an alternate account of non-individuality with the desired characteristics. I also claimed that the sort of assertions in physics that inspired the introduction of non-individuality talk are meaningful, but only when we recognize that words long used as count-terms are functioning as mass-terms. Fortunately, reinterpreting quantum terms as mass-terms requires a far more modest logical and semantic project than producing a theory of non-individuals. In fact, interpretations of quantum mechanics that treat references to particles in this way can already be found in the philosophical and scientific literature. For instance, Wallace and Timpson (2010) have elaborated a realist reading of quantum theory (viable in relativistic and non-relativistic contexts) that treats regions of space-time as fundamental—and conventionally individual—objects. Quantum particles or, more suggestively, quantities of discrete particle stuff, are properties of these regions. In quantum chemistry, an approach called 'Atoms in Molecules' or AIM is an important competitor to Density Functional Theory (Bader, 1990, 1991; Popelier, 2000). Broadly speaking, AIM treats the electron density function ρ (the square modulus of the many-electron wave-function) as the primary theoretical entity from which all chemically relevant properties can be derived. More specifically, atoms, molecules, and chemical bonds can all be defined in terms of geometric features of the gradient field on ρ. Atoms, for instance, consist of the union of an attractor (a point at which field lines converge) and its basin (the region of space from which the convergent field lines originate). In other words, the properties traditionally attributed to particles—whether individuals or not—are treated as quantities of stuff belonging to a spatial region. Adopting a mass-term semantics for quantum particle talk is thus perfectly compatible with drafting serious and intelligible interpretations of quantum physics.

Whatever full interpretation is ultimately adopted, particle terms are mass-terms with minimal parts. In this sense, they are not so exotic. The breathless story of quantum mechanics forcing us into a brave new world of metaphysical possibility in which the very notion of self-identity fails to apply is surely full of sound and fury. I have argued that key terms in this story signify nothing. To insist on pursuing a discussion in terms with no clear meaning is more mysticism than metaphysics.

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