# *Peirce on the Method of Balancing 'Likelihoods'*

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***Abstract***
Framed as a critique of David Hume's analysis of miracles, Peirce offers a sustained argument against an approach to historical inference he calls the "Method of Balancing Likelihoods" (MBL). In MBL the posterior probability that a disputed historical event has occurred is computed on the basis of the prior probability of that event occurring and the probability that each purported witness of the event has given accurate testimony. Peirce's critique of this method is hierarchical: he denies that an objective probability obtains for the truthfulness of witness testimony. Conceding this point, he asserts that, even if such objective probabilities exist, it is implausible to believe that witnesses are independent of one another. Conceding the first two points, Peirce argues that the very sampling process inherent to history necessarily introduces a strong probabilistic dependence that makes MBL unreliable. Finally, irrespective of the success of his first three criticisms, Peirce argues that MBL can be shown by empirical means to fail as a reliable method of inference. I reconstruct this hierarchical critique from a handful of Peirce's manuscripts, and emphasize its continuing relevance for modern accounts of judgment aggregation.

*Keywords: Charles S. Peirce, Historical Inference, Likelihood, Miracle, Judgment Aggregation.*

## ***1. Introduction***

Judgments concerning the occurrence of unusual events in the past are frequently justified on the basis of an intuitive probabilistic procedure for historical analysis. Roughly, the prior probability that the event in question occurred is contrasted with the probability of obtaining the given historical testimony if no such event took place. Only if the latter is less than the former should the occurrence be accepted as probable. Charles Sanders Peirce, across a collection of manuscripts, provides the components for a substantial critique of this method which he calls the "method of balancing likelihoods"[2] (hereafter MBL) (CP 7.176).[3] Exemplified by Hume's famous argument against the occurrence of miracles, MBL has been praised, criticized, or reformulated by disputants on both sides of the miracle debate [see, e.g., (Fogelin 2003), (Earman 2000), or (Sobel 1991) respectively]. However, this disputed inferential technique is not limited to the consideration of miracles. MBL encompasses the analysis of all marvelous or singular events recorded in historical testimony, and it is with respect to the method in its most general form that Peirce objects.

Peirce's intention is neither to dismiss nor embrace historical marvels (miracles included) out of hand, but rather to insist that a valid method of inquiry be applied for determining the truth in each case. MBL is not a valid "leading principle"[4] (Peirce 1880, p. 16) when applied to historical testimony. Indeed, if the line of argument pursued by Peirce is taken to its conclusion, MBL must be recognized as positively self-defeating. Drawing on two complete[5] manuscripts [(CP 7.162–255); (Wiener, Peirce, and Langley 1947)] and one fragment (CP 6.522–6.547) from Peirce, I reconstruct and defend his thesis that, even when made consistent with the probability calculus, MBL is an inadequate technique for assessing the veracity of historical testimony regarding singular events. In doing so, I examine a previously overlooked portion of Peirce's work that motivates an interpretation distinct from prior analyses.[6] Peirce's critique is hierarchical; he denies that there is any such thing as the objective "veracity" of a witness. Conceding this point, however, it is the case that, even if objective distributions corresponding to witness veracity exist, it is implausible to suppose that these distributions satisfy the independencies required by MBL. Conceding the first two points, Peirce argues that the very sampling process inherent to history necessarily introduces a strong probabilistic dependence that makes MBL inapplicable. Finally, irrespective of the success of his first three criticisms, Peirce argues that, as an empirical fact, MBL has failed as a method of inquiry.

## *2. The Scope of the Method*

In its most general form, MBL is an algorithm for determining the probability pertaining to the occurrence of an unprecedented event on the basis of historical testimony. To use one of Peirce's own examples, MBL is supposed to determine the probability that Pythagoras did indeed have a golden thigh on the basis of the stories received from three ancient authorities (CP 7.176). A less extravagant but equally appropriate example from ancient Greece concerns the number of Persian troops massed at Cunaxa. Xenophon tells us that the Persian host under the Great King numbered approximately 1,200,000 (Xenophon 1998, p. 111). In either case, the problem for the modern researcher is to judge the probability that the event described in historical testimonies actually took place, approximately as described. Given the list of eminent authorities making the claim, what probability can we attribute to Pythagoras' golden thigh? How likely is it that the Persian King Artaxerxes II amassed such a vast army in 401 BC given that Xenophon tells us so? MBL is an algorithm or rule for inferring these probabilities.

For Peirce, MBL is first given popular voice in Hume's famous argument against miracles or, more accurately, Hume's argument to the conclusion that one can never infer the reality of miracles strictly from testimony to that effect. While Hume's discussion explicitly engages testimony with respect to miracles, it is, as Peirce notes, of broader applicability (Wiener, Peirce, and Langley 1947, p. 220–1). Indeed, Hume himself concedes that his remarks encompass all "prodigies to be found in *history*, sacred and profane," (Hume 1988, 101, emphasis added). Singular events of the past recorded in extant historical documents are the domain of MBL. Note, however, that MBL (Hume's argument included) has nothing to say with regard to the present. As Peirce notes, "[Hume] expressly limits the argument to history. What ought to be said to a directly experienced miracle is a question which he does incidentally touch upon; but it is aside from his main argument" (Wiener, Peirce, and Langley 1947, p. 221). I will take for granted that the evaluation of directly observed events is outside the purview of MBL and irrelevant to the arguments in this paper.

### *3. Hume's Statement of MBL: the Argument against Miracles*

In the essay "On Miracles," published in the *Enquiry* (1988), Hume argues that we ought to believe in the occurrence of miracles[7] on the basis of testimony only in the case that falsity of the testimony would be even more improbable than occurrence of the miracle it describes. The occurrence of a miracle is a matter of fact and so can be established only by appeal to experience. Because matters of fact admit of "all imaginable degrees of assurance, from the highest certainty to the lowest species of moral evidence" (Hume 1988, p. 101), such a proposition can only be ascribed a probability; it cannot be known with certainty. The probability attached to an event depends on the preponderance of evidence drawn from experience. Hume speaks of weighing the evidence as an arithmetical process in which "we must balance the opposite experiments, where they are opposite, and deduct the smaller number from the greater, in order to know the exact force of the superior evidence" (Hume 1988, p. 102). The case in which there is complete or nearly complete uniformity amongst experiments—all observations fall on one side—constitutes the greatest "proof" or certainty which matters of fact can afford.

Experience teaches us that human testimony tends in some degree to be indicative of the events it describes. However, testimony and witnesses are not of a uniform character, and we must account for this in ascribing credibility. The probability that a given piece of testimony reports the truth ranges from low to practical proof. With respect to miracles—violations of natural law in Hume's use of the term—past experience amounts to proof that such events do not happen. Given the above calculus of probability then, any given testimony can at most amount to proof that exactly balances—in Hume's scheme of adding and subtracting evidences—the proof against the miracle, and can therefore at best leave us totally uncertain regarding the occurrence of the miracle. In Hume's words, "no testimony is sufficient to establish a miracle, unless the testimony be of such a kind, that its falsehood would be more miraculous, than the fact, which it endeavours to establish; and even in that case there is a mutual destruction of arguments, and the superior only gives us an assurance suitable to that degree of force, which remains, after deducting the inferior" (Hume 1988, p. 105–6). Because proof (uniform experience of the veracity of a witness) balances proof (uniform experience against the occurrence of the miracle), only a miracle violating a less than certain generalization could be established as probable on the basis of testimony. Since a generalization that is less than certain—one for which exceptions have been observed—would not constitute a law, and since a miracle is a violation of a law, Hume's maxim seems to rule out entirely the possibility of establishing the occurrence of a miracle on the basis of testimony.

### *4. Peirce's Construction of a Rigorous MBL*

Peirce captures Hume's statement of MBL in a pair of sentences: "A wise man proportions his belief to the excess of cases in experience in which facts resembling a fact in question have been found true over the cases in which such facts have been found false. In all cases we must balance the opposite experiments" (Wiener, Peirce, and Langley 1947, p. 221). Talk of belief and evidences (or experiments) is clearly central to Hume's consideration of probability, and so Peirce argues that in order to make Hume's claims intelligible, we must take care in interpreting these terms. In particular, Peirce focuses his exegetical efforts on the claim that "A wise man . . . proportions his belief to the evidence" (Hume 1988, p. 101). In order that Hume's treatment of balancing individual witness testimonies or "evidences" one against another be compatible with the calculus of probabilities it is necessary to attribute a peculiar interpretation to the term "evidence." The manner in which this required interpretation is different from the normal sense of evidence—as a specific fact or experimental outcome—is important to Peirce's critique. Hume talks as if by "evidence" he means simply some fact or observation, but in order to make any sense of the proposed probability calculus, this cannot be the case. Rather, by an item of "evidence" Hume must, Peirce suggests, be understood as referring to a stable, long-run frequency or innate propensity. At the very least, evidence must be understood as a probability distribution over facts of a class rather than specific facts themselves.

Peirce illustrates this interpretation by way of an example. Suppose it is known that a single black ball and ten white balls have been placed into an otherwise empty urn. "This knowledge assures us that if the balls are well stirred up and then one be drawn out, looked at, and thrown back, and this be repeated again and again, indefinitely, and if, at regular intervals, as the drawing goes on . . . the ratio of *all* white drawings to black drawings *from the beginning* be ascertained, then the only value which each ascertained ratio, as compared with the last previously ascertained ratio, will not, on the whole, depart from more often that it will approach, is the value 10:1" (Wiener, Peirce, and Langley 1947, p. 221, emphasis in original). In other words, random draws from the urn will, with practical certainty, demonstrate limiting frequencies of 1/11 for black balls and 10/11 for white. It is not the draw of a single particular ball but rather the known distribution of balls in the urn that constitutes an evidence of the sort Hume requires, at least if evidences are to be combined like probabilities. As Peirce puts it, "[s]uch a known fact (as that ten white balls and one black ball are contained in the urn,) which assures us that under circumstances definitely related to that fact (the drawings being made as prescribed), a kind of result (the drawing of a white ball) definitely related to the same fact will occur with a definite frequency in the long run, is to be termed an 'evidence,' or 'item of evidence'" (Wiener, Peirce, and Langley 1947, p. 221).

By contrast, the interpretation of the term "belief" is fairly straightforward, if unusual. Given knowledge of an evidence (in the above peculiar sense) Hume can be seen as asserting that contemplation of any of the possible future results pertaining to that evidence each excite a particular feeling in the mind of the contemplator. As Peirce characterizes it, Hume's notion of "belief" is to be understood as "expectation" (Wiener, Peirce, and Langley 1947, p. 222) in something close to the modern sense of the term.

Making Hume's qualitative statement of MBL rigorous with respect to the mathematics of probability requires more than a careful interpretation of "belief" and "evidence." Peirce constructs the remainder of a rigorous formulation of MBL from Hume's qualitative method of balancing likelihoods in stages. He begins by considering the odds in favor of a particular event having occurred given a collection of $m$ "pro" statements indicating that it has and, and $n$ "con" statements suggesting it hasn't. These statements, or "arguments" as Peirce calls them, are to be understood as the outcomes of distinct rules of inference, or as the "testimonies" of distinct witnesses.[8] Each testimony is presumed to have a well-defined probability of providing a correct answer in the given situation. Additionally, each testimony is *unconditionally independent* of all the others; the probability that a given testimony is correct is the same irrespective of whether the other testimonies are correct or mistaken. Suppose the probability that the first *pro* testimony is correct is $p_1$, that of the second is, $p_2$, etc., up through $p_m$. In a parallel fashion, denote the probability that the first *pro* argument is in error by $q_1$ (= 1- $p_1$), that the second is in error as $q_2$, etc., up to $q_m$. For the testimonies *against* the occurrence of the event, suppose the probability that each of the *con* arguments is correct to be $q_{m+1}$, $q_{m+2}$, etc., up through $q_{m+n}$, and denote the corresponding probabilities that the *con* arguments are incorrect by $p_{m+1}$, $p_{m+2}$, etc. up through $p_{m+n.}$ [9] For any such set of testimonies, only one of two possibilities can obtain: either (i) all of the *pro* testimonies are correct, the event did occur, and the *con* testimonies are in error, or else (ii) all of the *con* testimonies are correct, the event did not occur, and those witnesses offering *pro* testimonies are in error. With the notation we have chosen, the probability that all of the testimonies in favor of the conclusion are correct and all of those opposed are in error is simply $p_1 \times p_2 \times \ldots \times p_{m+n}$. Similarly, the probability that all of the *con* arguments give the right answer while all of the *pro* get it wrong is $q_1 \times q_2 \times \ldots \times q_{m+n}$. The odds in favor of the conclusion (in favor of the event having actually occurred) are then simply the ratio of these two probabilities[10]:

$$O(\text{conclusion}) = \frac{p_1 \cdot p_2 \cdot \dots \cdot p_{n+m}}{q_1 \cdot q_2 \cdot \dots \cdot q_{n+m}}.$$

Note that here and throughout this paper, odds ratios are indicated by $O(x)$ and probabilities by $P(x)$.

If, to capture Hume's talk of adding and subtracting favorable and unfavorable arguments or experiments, "we suppose that the impression made on the mind of the wise man is proportional to the logarithm of the *odds* as its exciting cause, then the total impression will be" (CP 7.165, emphasis in original):

$$\log\left(\frac{p_1 \cdot p_2 \cdot \dots \cdot p_{n+m}}{q_1 \cdot q_2 \cdot \dots \cdot q_{n+m}}\right) = \log\left(\frac{p_1}{q_1}\right) + \log\left(\frac{p_2}{q_2}\right) + \dots + \log\left(\frac{p_{n+m}}{q_{n+m}}\right).$$

Thus, the degree to which we should assent to the truth of a conclusion (or the occurrence of an event) is additive in the sense that each testimony with better than 50:50 odds of getting the right answer increases our confidence while each additional testimony with less than 50:50 odds of being right reduces it.

This first theory is what Peirce calls "Hume's Theory Improved" (HTI), and it is this formalization of MBL which previous commentators have considered when reconstructing Peirce's argument (Merrill 1991). However, HTI is only the first stage of Peirce's construction of a rigorous MBL in terms of the probability calculus. His strongest and most general criticism against the method cannot be understood without having before us the strongest version of MBL which Peirce puts forward. For this reason, I will diverge from prior treatments of Peirce's essay and explicate the remainder of his formal reconstruction before interpreting Peirce's criticisms. In presenting Peirce's formalism in this section, I will modify his examples for clarity of exposition. However, since an exegesis of this portion of his work has not been published elsewhere, I will indicate through footnotes how my examples can be mapped onto Peirce's originals.

To continue his construction of MBL, Peirce correctly notes that there is a mistake in the HTI computation made above. The expression $p_1 \times p_2 \times \ldots \times p_{m+n}$ does not represent the probability of the event having occurred given the testimony, but rather the probability that the first $m$ witnesses are correct and the last $n$ wrong, *irrespective of what they say*. If we attempt to correct for this oversight, however, we discover that, for the analysis to work, a particular independence must hold that is not the independence we assumed. To show that this is the case, Peirce constructs a somewhat stilted example involving the drawing of boxes from an urn, wherein each box contains either gold or lead.[11] We can instead continue to speak of witnesses and their testimonies. Suppose we have testimony from two witnesses, call them $w_1$ and $w_2$, indicating whether or not some proposition $q$ is the case ($q$ could, for instance, be the claim that a particular miracle occurred). Peirce requires (as does Hume) that there be some definite probability attached to this proposition. For his example, Peirce takes $P(q)$ to be 3/7 (and thus $P(\neg q) = 4/7$). As an objective fact of the matter, let us further assume that the probability of $w_1$ providing accurate testimony (an event I'll notate as $w_1+$) is 9/14, and similarly $P(w_2+) = 3/4$. Both witnesses provide independent testimony such that $w_1$ is as likely to be right when $w_2$ is right as when $w_2$ is wrong, and vice versa. To demonstrate the failure of the naïve HTI derived above, Peirce constructs a table[12] of joint probabilities consistent with these assumptions (CP 7.168). An adaptation of it is presented as Table 1 below, in which the numbers represent objective probabilities.[13]

**Table 1.**

| | $q$ | | $\neg q$ | |
|---|---|---|---|---|
| | $w_1$ says "$q$" | $w_1$ says "$\neg q$" | $w_1$ says "$q$" | $w_1$ says "$\neg q$" |
| $w_2$ says "$q$" | 15/168 | 35/168 | 14/168 | 6/168 |
| $w_2$ says "$\neg q$" | 21/168 | 1/168 | 10/168 | 66/168 |

It is straightforward to verify that the testimonies of the two witnesses, as represented in the table, are independent of one another. Using the numbers in Table 1, the probability of witness 1 being right, notated $P(w_1+)$, is simply 108/168 or 9/14. This is also the case if we condition on the accuracy of the second witness' testimony: $P(w_1+|w_2+) = 81/126 = P(w_1+|w_2-) = 27/42 = P(w_1+) = 9/14$, where $w_2$- indicates that the testimony of $w_2$ is false. Thus, the two distributions are unconditionally independent, as required by HTI.

If HTI is applicable, we would expect the odds that both experts are correct to be given by the product of the odds that each is correct: (3/1)(9/5) = 27/5. That this is the case is readily verified from the table: $w_1$ and $w_2$ correctly agree that $q$ or $\neg q$ is the case 15 times and 66 times respectively for every 14 + 1 times they agree and are wrong. This of course results in odds of 81/15 = 27/5. However, when working directly from the table in this fashion, it becomes clear that the odds computed are *not* the odds in favor of *q*, but rather the odds that all witnesses both agree and are correct, irrespective of what they claim to be the case. Additionally, the calculation ignores information at hand: it is known what each witness claims, not just the probability that he is right. What one wants to know, if HTI is to be made relevant to the question Hume considers, is whether a given event occurred given testimony that it did or did not do so. In this example what is required is the probability that *q* is the case given that both witnesses claim it is (i.e. that both arguments are *pro*), not the probability that all witnesses agree. However, once we have identified the odds we should be calculating and have accounted for all of the information that may be brought to bear (i.e. the specific claims of the experts), it becomes clear that the naïve theory of HTI fails because the wrong independencies have been assumed.

To see this, Peirce suggests that we try applying HTI with odds conditioned on our knowledge of what each witness claims rather than entirely unconditioned. Referring again to Table 1, we can see that, assuming $w_1$ declares that $q$ is the case, the odds that $w_1$ is right are (15+21)/(14+10) = 3/2. On the other hand, given that $w_2$ asserts $q$, the odds that $w_2$ gets it right are (15+35)/(14+6) = 5/2. Note that the two experts are still independent of one another, conditional either on what $w_1$ actually says or on what $w_2$ says: $O(w_1+ \mid w_1$ says "$q$", $w_2+) = O(w_1+ \mid w_1$ says "$q$", $w_2-) = 3/2$. However, if one were to attempt to compute the probability of the $q$ being the case by simply multiplying the odds of the two experts being correct (conditional now on what $w_1$ says or on what $w_2$ says), one would arrive at odds of (5/2)(3/2) = 15/4. Looking at Table 1, these odds are plainly in error: $O(q \mid w_1$ says "$q$", $w_2$ says "$q$") = 15/14. Upon recognizing this inconsistency, Peirce suggests that one might be inclined to correct this shortcoming by including the antecedent odds that $q$ is the case (= 3/4) in the product. However, doing so gives a value of (3/2)(5/2)(3/4) = 45/16 which is still incorrect.

What has gone wrong, as suggested above, is that we have failed to compute the relevant conditional probability. Though he gives only an abbreviated summary of the results, the final stage of Peirce's reconstruction introduces an expression that does in fact represent the relevant conditional probability, namely the probability of $q$ being the case given the testimonies from $w_1$ and $w_2$ (CP 7.168). However, since Peirce's final expression of MBL is stated in terms of odds ratios without much hint at a derivation, I will work forward from the given distributions using the modern probability calculus, rather than attempt to work backward from what appears in his text. To begin with, we can write in the notation introduced above an expression for the probability that $q$ is the case, conditional on the testimonies of both witnesses to that effect:

$$P(q \mid w_1 \text{ says "}q\text{"}, w_2 \text{ says "}q\text{"}) = \frac{P(w_1 \text{ says "}q\text{"}, w_2 \text{ says "}q\text{"} \mid q)P(q)}{P(w_1 \text{ says "}q\text{"}, w_2 \text{ says "}q\text{"})}$$
$$= \frac{P(q)P(w_2 \text{ says "}q\text{"} \mid q)P(w_1 \text{ says "}q\text{"} \mid q, w_2 \text{ says "}q\text{"})}{P(w_1 \text{ says "}q\text{"}, w_2 \text{ says "}q\text{"})} = \frac{P(q)P(w_2+ \mid q)P(w_1+ \mid q, w_2+)}{P(w_1 \text{ says "}q\text{"}, w_2 \text{ says "}q\text{"})}.$$

If we assume, that the veracity of each witness is independent of the other conditional on the truth of $q$, then $P(w_i+ \mid q, w_2+) = P(w_i+ \mid q)$ and

$$P(q \mid w_1 \text{ says "}q\text{"}, w_2 \text{ says "}q\text{"}) = \frac{P(q)P(w_2+ \mid q)P(w_1+ \mid q)}{P(w_1 \text{ says "}q\text{"}, w_2 \text{ says "}q\text{"})}. \tag{1}$$

Equivalently, the odds in favor of $q$ being the case, given that both witnesses say so, is given by:

$$O(q \mid w_1 \text{ says "}q\text{"}, w_2 \text{ says "}q\text{"}) = \frac{P(q)P(w_2+ \mid q)P(w_1+ \mid q)}{P(\neg q)P(w_2- \mid \neg q)P(w_1- \mid \neg q)}. \tag{2}$$

Equation (1) can be generalized to provide an expression for computing the probability of the occurrence of an event (indicated by $q$) on the basis of $n$ independent testimonies:

$$P(q \mid w_1 \text{ says "}q\text{"}, w_2 \text{ says "}q\text{"}, \ldots, w_n \text{ says "}q\text{"}) =$$
$$\frac{P(q)P(w_1+ \mid q)P(w_2+ \mid q)\cdots P(w_n+ \mid q)}{P(w_1 \text{ says "}q\text{"}, w_2 \text{ says "}q\text{"}, \ldots, w_n \text{ says "}q\text{"})}. \tag{3}$$

While Peirce does not provide a derivation like that above, he does give a final expression for the odds in favor of $q$ being the case given that both witnesses say so:

$$O(q \mid w_1 \text{ says "}q\text{"}, w_2 \text{ says "}q\text{"}) =$$
$$\frac{P(w_1+, w_2+, q)}{P(w_1-, w_2-, \neg q)} = \frac{P(w_1+, w_2+ \mid q)P(q)}{P(w_1-, w_2- \mid \neg q)P(\neg q)}. \tag{4}$$

He also provides exact conditions under which "the required independence" (CP 7.186) may be found[14]:

$$P(w_1+, w_2+|q) = \frac{P(w_1+, w_2-|q)P(w_1-, w_2+|q)}{P(w_1-, w_2-|q)},$$

$$P(w_1-, w_2-|\neg q) = \frac{P(w_1+, w_2-|\neg q)P(w_1-, w_2+|\neg q)}{P(w_1+, w_2+|\neg q)}.$$

With a little algebra, these conditions can be reduced to $P(w_1+ \mid w_2+, q) = P(w_1+\ w_2 \mid -, q)$ and $P(w_1+ \mid w_2+, \neg q) = P(w_1+ \mid w_2 -, \neg q)$, exactly those assumed in deriving Equation (1). If these conditions hold, then Equation (4) reduces to Equation (2). If we take Equation (2) (or equivalently Equation (3)) to be the final formalization of MBL, "[t]hen, when the essential [independence] conditions are fulfilled, this method is perfectly correct" (CP 7.169).

As an example for which MBL is applicable, Peirce offers the distributions given below in Table 2. Here $P(w_1+ \mid w_2+, q) = 21/24 = P(w_1+ \mid q) = 35/40 = 7/8$, and so the requisite independencies hold. Additionally, unlike for the distributions of Table 1, it is true that the odds of $q$ being the case are given by Equation (2), since

$$\frac{P(q)P(w_2+\mid q)P(w_1+\mid q)}{P(\neg q)P(w_2-\mid \neg q)P(w_1-\mid \neg q)} = \frac{\left(\frac{40}{70}\right)\left(\frac{35}{40}\right)\left(\frac{24}{40}\right)}{\left(\frac{30}{70}\right)\left(\frac{25}{30}\right)\left(\frac{12}{30}\right)} = 21/10.$$

It is evident from Table 2 that this is the correct value for $O(q \mid w_1$ says "$q$", $w_2$ says "$q$").

**Table 2.**

| | $q$ | | $\neg q$ | |
|---|---|---|---|---|
| | $w_1$ says "$q$" | $w_1$ says "$\neg q$" | $w_1$ says "$q$" | $w_1$ says "$\neg q$" |
| $w_2$ says "$q$" | 15/168 | 35/168 | 14/168 | 6/168 |
| $w_2$ says "$\neg q$" | 21/168 | 1/168 | 10/168 | 66/168 |

It will be useful to recap the corrections to HTI that resulted in the rigorous, consistent version of MBL against which Peirce directs his criticisms:

**MBL**: Given $n$ witnesses $w_1, w_2, \ldots, w_n$ providing testimonies that $q$ is the case, where the probability for each witness being correct conditional on the occurrence of the event is known and independent of every other witness such that $P(w_i+ \mid q) = P(w_i+ \mid q, w_j+)$ for all $i \neq j$, the probability that $q$ is the case is given by Equation (3).

Before moving on to Peirce's criticisms of MBL, it is worth emphasizing two points. First, he has introduced out of mathematical necessity a distinct term for the antecedent probability of the proposition in question. This term does not appear in HTI as outlined above or, for that matter, in De Morgan's essay on amalgamating testimonies (1849). In Hume's qualitative computation, the extraordinarily low prior probability of miracles is the figure against which the testimony in question is being weighed. Thus, Peirce's corrected theory of MBL is closer to Hume's qualitative text in this regard than was the so-called HTI. Second, Peirce grants that MBL, *not* HTI is applicable within certain circumscribed conditions. However, it is only HTI that previous commentators (Merrill 1991; Legg 2001) have considered when reconstructing Peirce's criticisms. Since Peirce goes on to correct HTI, it is clearly not what he has in mind when condemning Hume's argument. Thus, it is essential to consider the final version of MBL when asking why the method is inapplicable in the case of historical testimony.

### *5. What's Wrong with MBL?*

Peirce offers both a hierarchical critique of MBL in its own terms (on the basis of considerations of probability) and a straightforward empirical attack that is independent of the theoretical justification for the method. Taking each of these in turn, we begin with the critical analysis of the rigorous statement of MBL provided in the preceding section. It is important to note that, for MBL to even get off the ground, we must overlook an immediate and serious difficulty: historical testimonies do not in themselves constitute "evidences" of the sort required by Hume. Each is an instance, not a distribution indicating the "veracity" of a witness. As Peirce puts it, "The single instances though they may be evidences in the ordinary sense of the word are not 'evidences', in the sense [Hume's] argument requires" (Wiener, Peirce, and Langley 1947, p. 222). This in itself is a serious objection. Let us suppose, however, that we have some independent access to such distributions[15] for any given witness, in addition to the particular testimonies regarding some event. That is, let us assume that testimonies are samples drawn from some known distributions. Then, says Peirce, we still have to face a series of challenges to the rigorous form of MBL as it is applied to history.

To begin with, it is not clear that the objective distributions required by MBL exist. In "The Logic of History" (CP 7.162–255), Peirce provides a sustained argument to the conclusion that, with respect to testimonies, "the inappropriateness of the application of the conception of probability. . .is striking" (CP 7.178). Peirce's notion of an objective probability requires for its application the satisfaction of a number of physical conditions. Specifically, the "calculus of chances" (CP 7.178) provides an appropriate description only in the case of a class of phenomena (such as throws of a die) in which myriad indiscernible causes conspire to produce the outcome in such a way as to ensure stable, long-run frequencies. In Peirce's words, "[i]n playing a game, say with dice, there is this good reason for the calculation of chances, that any one face turns up as often as any other, quite independently of the result of any other throw, and the cause of the die turning up any particular face at any particular throw is quite beyond our powers of analysis" (CP 7.178). To ensure objective probabilities, the example Peirce actually employs in the development of MBL does not involve two generic witnesses declaring the truth or falsity of some proposition $q$ as my example did. Instead, he imagines two "experts" whose testimonies concerning a particular proposition are artificially tied to a fixed stochastic process, similar to flipping a coin. The probability that one of these experts tells the truth has nothing to do with the nature or personality of the expert, but rather depends on the stochastic process linking a particular physical property he observes to the physical state of affairs he affirms. In the case of genuine witness testimony, however, there is no confluence of unknown causes under repeatable conditions—the specific situation dictates his judgment, and the situation cannot *in principle* be replicated to make sense of the long-run frequencies essential to the notion of probability: "[T]ake a question of history. We do not care to know how many times a witness would report a given fact correctly, because he reports that fact but once. If he misstates the matter, there is no cooperation of myriad causes. It is on the contrary due to someone cause which, if it cannot often be ascertained with certainty, can at any rate be very plausibly guessed in most cases, if the circumstances are closely inquired into. . ." (CP 7.178). Peirce explicitly denies a stable long-run "veracity" for a witness.

Peirce's criticism here may be reasonably interpreted as the claim that historical testimonies of the relevant sort are *sui generis*, and there can be no appropriate reference class. An individual cannot be asked to judge of precisely the same historical event in the same context more than once, because that context (the individual's knowledge, past experiences, etc.) are different.[16] There is no sample space of judgments from which we can in principle draw an indefinite number of samples. While we might be able to tally up a given person's true or false judgments, "[a] mere general ratio of true statements to false, would be utterly insufficient. . .even if it really existed" (CP 7.178). The distributions required cannot be had. Peirce is not asserting that witness behavior is not law-like, but rather that it is law-like in such a fashion as to make the application of probabilistic concepts moot.

As I have claimed already, Peirce's critique is hierarchical: even if we concede that well-defined probability distributions obtain for human witnesses—of the sort required in Equation (3)—the requisite independencies generally will not obtain. Though "[c]ircumstantial evidences are. . .often sufficiently independent" (CP 7.176), this is seldom the case with direct testimony. Witnesses, particularly those inclined to write historical tracts, are strongly influenced by one another in ways that result in negative or positive correlations. "The same circumstances which lead one witness into error are likely to operate to deceive another" (CP 7.176). Witnesses defer to one another's authority or, just as often, contradict one another out of a variety of motives. It is implausible to suppose that a witness' testimony is independent of all others, conditional on the fact of the matter. Furthermore, Peirce insists that "[t]he method of balancing likelihoods not only supposes that the testimonies are independent but also that each of them is independent of the antecedent probability of the story; and since it is far more difficult to make allowance for a violation of this requirement than of that of the independence of testimonies, it becomes a much more serious matter" (CP 7.176). It is important to note that Peirce is speaking here of independence with respect to the *probability* of the occurrence of the event, not with respect to the occurrence of the event itself. The probability calculus is not equipped to handle this possibility. The expression is not a well-formed formula in the probability calculus—there is no way to evaluate the expression. Yet testimonies recorded as history—particularly those reporting the marvelous—are often, if not as a rule, recorded precisely because the event described was unusual: "we may almost say that ancient history is simply the narrative of all the unlikely events that happened during the centuries it covers" (CP 7.176). The lower the antecedent probability of an event, the more likely it is to attract testimony upon its occurrence. If testimony is not independent of the probability of the event, then Equation (3) does not follow and MBL cannot be brought to bear on the question.[17] Thus, MBL requires two distinct and highly suspect independencies without which it and Hume's argument are inapplicable.[18]

To make matters worse for MBL, if it is granted that witnesses are generally independent of one another and their testimony is independent of the antecedent probability of the event described, the very method of sampling implicit in historical testimony would compromise this independence. This is, I would argue, the objection Peirce has in mind when he says that Hume "has completely mistaken the nature of the true logic of abduction" (CP 6.537). It is this, the strongest objection Peirce offers against the application of MBL to historical testimony, which previous commentators have overlooked. To understand this final objection requires a brief digression into Peirce's philosophy of science.

The scientific process as Peirce describes it consists of three conceptually distinct and methodologically independent stages. The first stage he refers to as "*abduction*" (CP 6.525), the second as "*deduction*" (CP 7.203), and the third as "*induction*" (CP 7.206). The first stage involves the probationary adoption of viable explanations or hypotheses. Abduction is the process of "adopting a hypothesis as being suggested by the facts" (CP 7.202). A "*hypothesis*" (Buchler 1955, p. 151) is a proposition that asserts something about the nature of a particular object, about properties of a class of objects, or about the world in general, and that explains the facts observed. To say that a hypothesis explains the facts observed is to assert that, were the proposition true, the observed facts would be a likely result. Abduction, or the formation of an explanatory hypothesis, is thus an inference with the following form:

(1) Surprising fact C is observed.
(2) If proposition A were true, C would be likely.
___
(3) A is an explanatory hypothesis for C.

As Peirce stresses, not every well-formed proposition is a viable hypothesis. Abduction is a filter which picks out those propositions which constitute explanations worthy of test. When a hypothesis is abductively valid, suggests Peirce, it warrants significantly greater confidence after its predictions are verified than would be the case for an arbitrarily selected proposition.

The latter two stages of scientific inquiry consist of deducing consequences of a given hypothesis and testing these consequences against fresh observations. While many of the details of deduction and induction are irrelevant to his argument against MBL, an essential point for Peirce is that hypothesis generation and testing cannot make use of the same facts; whatever observations provide the basis for adopting a hypothesis cannot serve to verify that hypothesis. If, contrary to this dictum, the same facts were used to justify both the claim that a hypothesis is explanatory of the facts and that the hypothesis is true, then the justification of hypothesis acceptance would be circular. More particularly, if one attempts to apply any sort of statistical methods of inductive inference, the data originally used to form the hypothesis under test represent a biased sample—they necessarily confirm the hypothesis constructed as an abductive inference from the same data. While he does not state this explicitly, this is presumably the motivation for Peirce's insistence on both the logical separation of abduction from the stages that follow as well as the demand that only facts not under consideration in the abductive stage be used in the inductive stage.

Turning our attention back to the particular case of MBL, the difficulty is as follows. The hypothesis that an unusual (or miraculous) event did in fact occur is an abductive inference from the set of historical testimonies to that fact. If that same set of testimonies is then used to *test* the hypothesis that the event occurred, one is making use of a biased sample—only confirmatory evidence has been selected, rather than a random sample of all historical judgments. It is doubtful whether or not such a random sampling process is even possible, but insofar as the same set of testimonies is used in the abductive and inductive stages, the method is invalid. As Peirce puts it, were it the case that we could treat these testimonies as independent samples, then "a man who merely knew of a certain urn of balls that a hundred white drawings had been made from it, would, in the absence of all information in regard to the black drawings, be entitled to a definite intensity of 'belief' in regard to the next drawing, and not only so, but the degree of this 'belief' would remain quite unaffected by the further information that the number of black drawings that had ever been made from the urn was zero" (Wiener, Peirce, and Langley 1947, p. 223). When seen in terms of abduction, the complaint is as follows. Knowing that a certain number of white drawings had been made from the urn supports the abductive inference that the urn contains some number of white balls. However, this same knowledge by itself cannot be used to estimate the relative proportion of white balls in the urn or, derivatively, the probability that the next one drawn will be white. This is roughly what MBL suggests can be done but this is to ignore the fact that the white drawings under consideration were all, in a sense, selected for consideration because they were white drawings. They do not constitute a representative sample of random draws.

Things are actually more complicated in the case of MBL because we are considering a collection of probability distributions, one for each witness issuing testimonial evidence, rather than just a collection of events or instances. But the analogous problem remains. If, in a computation like that shown in Equation (3), we consider only those testimonies (and associated distributions) from witnesses who claimed an event occurred rather than the full set of testimonies from viable witnesses, the computation is driven entirely by a subclass of witnesses whose testimony cannot be independent—by virtue of our selection process, the probability that one claims an event happened given that another in the set also claims it is unity.

When the logic of scientific inquiry is ignored, a strong and ineliminable selection bias is introduced which undermines the validity of MBL. In Peirce's terms, it is logically circular to use the very testimonies from which the miracle hypothesis is abductively inferred to further justify that hypothesis inductively. Peirce expends a great deal of effort constructing a mathematically rigorous version of MBL in which the probability of an event is correctly computed conditional on the occurrence of testimonies displaying the requisite independencies. He does not, as Merrill suggests,[19] condemn outright the use of the calculus of chances in the analysis of human testimony or judicial evidence. He only emphasizes that MBL is supported by the "doctrine of chances" under restrictive conditions that "are not even roughly fulfilled *in questions of ancient history* . . ." (CP 7.170, emphasis added). What he objects to is the application of Equation (3) to the analysis of historical testimony, and his strongest objection concerns the use of one and the same fixed sample of testimonies in both the abductive and inductive stages of scientific inference.

Finally, if all of Peirce's arguments on the basis of his formal reconstruction fail, if all three of the difficulties outlined above can be overcome, at least in theory, Peirce has at his disposal one more important critique. The efficacy of MBL as a rule of inference is itself open to empirical investigation. If the rule is valid then it should, in a majority of cases, lead to the correct judgment regarding the occurrence or non-occurrence of singular events described in historical testimony. From this perspective, Peirce argues that those scholars employing MBL to judge historical texts "were found to be more or less fundamentally wrong in nearly every case, and in particular that their fashion of throwing all the positive evidence overboard in favor of their notions of what was likely, stands condemned by those tests" (CP 7.182). The discovery of the city of Troy is such a prominent case—critics on the basis of MBL (at least informally) dismissed accounts of the Trojan War as entirely fictive, when this was not the case. Of course, anecdotes are insufficient here and Peirce does not bring an appropriately large, randomized sample to bear on the issue. Nonetheless, this final assault on MBL stands to be decisive if proper experiments or analyses of extant data are carried out.

Supposing MBL is inapplicable to historical testimony, what does Peirce offer in its stead? The bulk of his manuscript, "The Logic of History" (CP 7.162–255), is dedicated to outlining the proper application of the stages of scientific inference—abduction, deduction, and induction—to the problem of historical testimony. Rather than attempt to use historical testimony as the basis of an inductive verification of the content of that same testimony, we ought instead to seek abductively valid hypotheses that render unsurprising *all* of the available evidence, including the testimony. Each such hypothesis should then be inductively tested by deducing and verifying observable consequences. Of course, the sort of induction open to us in such a case cannot assign objective probabilities to the conclusion, but can nonetheless increase our confidence (this is an induction over consequences of a proposition, not samples from a determinate class). Peirce's positive account is well summarized in (Legg 2001).

### *6. Implications of the Argument*

Peirce's argument as I have presented it is more than a specific refutation of Hume's method of inference concerning miraculous occurrences. Peirce saw Hume as presenting a method of inference applicable to all historical testimony. Viewed even more expansively, MBL is a general method for assessing confidence in an aggregation of testimonies or judgments. Elements of Peirce's critique, stated in the most general terms, are as germane to modern theories of judgment aggregation as they were to the work of historians contemporary to Peirce.

Take for instance, modern models of decision-making such as models of jury decisions. Classically, the problem of jury decisions concerns the probability that a collection of jurors correctly decide a case, assuming there is an objective fact of the matter. This problem is in many ways similar to the central problem considered in this essay: the judgment of each juror is akin to the testimony of each "expert" or each historical witness, and the probability that the jury decides correctly is analogous to the probability that an event took place, given testimony to that effect. The Marquis de Condorcet published an account of this jury problem in his 1785 treatise on probability (Boland 1989). Like MBL, the Condorcet Jury Theorem, which gives the probability that a majority decision of the jury is correct, assumes independence between juror decisions and between juror decisions and the antecedent probability of guilt (Boland 1989). Accounting for violations of at least the first of these independence conditions is a current focus of research on jury decision models (Dietrich and List 2004).

Not restricted to history or juries, Peirce's concerns are relevant to judgment aggregation at large. For instance, in a paper that considers the merger of "belief bases"—collections of propositional judgments—from multiple agents, Pigozzi and Hartmann (2007) compute the probability that a particular method of combining sets of propositions ranks the correct set first. In their calculation, it is assumed that the probability of any given agent choosing one of a binary pair of propositions is equal and, implicitly, independent of the choices of other agents. This is precisely the pair of assumptions Peirce criticized with respect to applying MBL. These concerns extend to judgment aggregation construed broadly. In this domain, one is typically attempting to determine the probability that an aggregate proposition is true as a function of the probabilities that each individual agent makes true judgments. As a rule, judgments are not likely to be independent of one another, though this is perhaps approximately satisfied at times. Additionally, realistic models must acknowledge that the judgments of individual agents are likely to be dependent on the antecedent probability of an event in many, if not most, circumstances. Perhaps most importantly, one must worry about the sort of selection bias which Peirce presented as a confusion of abductive and inductive inferences. When agents are free to adopt whatever belief bases they choose, the problem of assessing the truth of a shared proposition is quite similar to that of assessing the truth of repeated historical testimonies—in asking for the probability of a proposition believed by a number of agents, it is necessary to make use of a fresh sample of agents if one is to avoid introducing strong probabilistic dependencies. This particular consideration appears to be unique to Peirce's work, as is the unified hierarchical critique of MBL outlined above.

It is because of its broad applicability to modern research programs that Peirce's analysis deserves closer inspection and wider consideration amongst those concerned with discerning truth from testimonies or with aggregating judgments in general. While his attack on Hume is interesting in its own right, the argument, like much of Peirce's work, carries implications broader than the restricted thesis it sets out to establish.

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## NOTES

1. I would like to thank Teddy Seidenfeld, Steve Fancsali, and two anonymous reviewers for the *Transactions of the Charles S. Peirce Society* for helpful discussion and comments on previous versions of this paper.

2. This is the label Peirce gives to the inferential method which he attributes to Hume, and which I consider here. Peirce generally treats "likelihood" as

synonymous with "nothing more than the expression of our preconceived ideas" (Wiener, Peirce, and Langley 1947, p. 225), and does not intend it in the technical sense in which it is used in the modern literature of probability and statistics.

3. Throughout this paper I use the notation (CP x.xxx) to cite material from the *Collected Papers of Charles Sanders Peirce* (1935). In this convention, (CP 1.234) for instance refers to volume 1, paragraph 234.

4. A "leading principle" is a "habit of thought" which determines the passage from a set of judgments (the premises) to a consequent judgment (the conclusion) (Peirce 1880, p. 16). The notion is akin to an inference rule in formal logic, though for Peirce the rule is not merely syntactical.

5. So far as I know, Merrill (1991) is the only other author to have considered these two manuscripts together as advancing a single coherent argument.

6. While both Merrill (1991) and Legg (2001) each remark on a subset of the criticisms examined in this paper, neither reconstructs Peirce's rigorous version of MBL, and so both miss Peirce's strongest objection. So far as I am aware, no explication of Peirce's full development of MBL in the probability calculus appears in the literature. This formal account of MBL, together with Peirce's insistence (in CP 6.522–6.547) on the relevance of abductive logic to the evaluation of MBL, supports an interpretation of Peirce that differs from—and in some respects directly contradicts—the views of Merrill and Legg.

7. In this essay, Hume defines a miracle to be "a violation of the laws of nature" (Hume 1988, p. 104). However, Peirce notes that Hume's argument can make do with a less stringent notion of miracle: "A miracle is an event against which there is uniform experience. . . . This is the only definition of a miracle which is pertinent to the argument" (Wiener, Peirce, and Langley 1947, p. 224). It is this weaker notion of miracle that Peirce assumes throughout his criticisms.

8. At the beginning of his derivation of HTI, Peirce cites a paper of De Morgan's (1849) in which he applies the probability calculus to testimony. In the section of this paper that deals with weighing evidence, De Morgan distinguishes between testimonies and arguments. Specifically, he says that "*Argument* is an offer of proof, and its failure only a failure of proof: the conclusion may yet be true. *Authority* is an offer of testimony, and its failure is a failure of truth: nothing can furnish absolute reason for distrusting the authority on future occasions except the proof that the conclusion asserted is false" (De Morgan 1849, p. 393) (emphasis in original). He elaborates on the notion of authority: letting "$\mu$ being the chance that an assertion of an individual, made on the best of his knowledge and belief, is true, I shall call $\mu$ the value of his testimony." When Peirce uses the term "argument" it is exactly this latter notion of authority he has in mind, *not* what De Morgan calls "argument." This inversion of De Morgan's terminology may be the source of the following criticism reported by Peirce: 'In the ordinary text books on the Doctrine of Chances, so much of this theory as is given at all is only given in their chapters on the probability of testimony; and I will mention that Professor F.Y. Edgeworth says that in extending it to all independent arguments that have definite general probabilities I am "confusing" testimonies with arguments' (CP 7.168).

9. Using $p$'s to represent chances of error for the con arguments—contrary to the convention for pro arguments—simplifies later expressions and is true to Peirce's presentation.

10. Though the final expression I have derived for this odds ratio is identical to that in Peirce (CP 7.165), I have altered the interpretation of his notation slightly

in order to present a more transparent demonstration. For a more faithful if not verbatim presentation of this portion of Peirce's reconstruction, see (Merrill 1991, pp. 96-98).

11. The actual example as Peirce presents it involves an urn filled with boxes, each of which contains a sample of either gold or lead. A given sample of metal is either yellow or gray, heavy or light. More often than not, the samples that are actually gold are heavy and yellow, but there are some boxes containing gold that is light and gray, or heavy and gray, etc. What matters is that there is a definite proportion of each sort of sample in the urn. If repeated draws are made at random from the urn (with replacement), then there is a definite probability that, given that a box contains a heavy metal, it contains gold (or given that it contains a yellow metal, it contains gold). The witnesses in Peirce's example are two "experts," one of which infallibly declares a metal sample to be gold if it is heavy, the other says "gold" if the sample is yellow. To translate my example into Peirce's, $w_1$ is to be identified with the "material-expert," $w_2$ with the "color-expert," and the proposition $q$ should be read as "the box contains gold."

12. Peirce's tables contain only whole numbers representing the number of boxes of each type present in the urn from which draws are made (they are *not* indicative of some particular finite sample of draws). To get the probabilities shown in my tables, simply divide Peirce's numbers by the total number of boxes in the urn.

13. For Peirce, the mechanical drawing of boxes from an urn is an antecedent condition to which a proper probability attaches (see (Buchler 1955, Ch. 12)). This is presumably why he chooses such a process to underlie his witness testimony in constructing what he argues to be a viable version of MBL. He will argue, however, that actual human testimony, divorced from such artificial processes as infallibly reading off colors, cannot have an objective probability associated with it.

14. While Peirce correctly gives the conditions for independence, he passingly refers to the ratio $P(w_1 \text{ says "}q\text{"}|\text{q})/P(w_1 \text{ says "}q\text{"}|\neg\text{q})$ as the odds that $q$ is true given that $w_1$ says so and the ratio $P(w_2 \text{ says "}q\text{"}|\text{q})/P(w_2 \text{ says "}q\text{"}|\neg\text{q})$ as the odds that $q$ is true given that $w_2$ says so. Given that his final expression for use in MBL is also correct, this misattribution can be safely ignored.

15. In order to apply MBL, one needs knowledge of the collection of distributions appearing in Equation 3. As Peirce was aware, finite samples of each witness' true and false testimonies could at best provide uncertain estimates of these distributions. One would then have to take care in accounting for this extra degree of uncertainty in a manner not done in the construction of MBL.

16. Legg (2001) offers a similar interpretation of this portion of Peirce's "The Logic of Drawing History from Ancient Documents."

17. While Legg (2001) and especially Merrill (1991) emphasize Peirce's criticism on grounds of independence, neither provides an account of which independencies matter and why. Certainly HTI (recounted by Merrill but not Legg) assumes that the probability of the truth of a conjunction of testimonies is given by the product of the probabilities of the truth of each testimony, and this assumption requires the unconditional independence of testimonies from one another. However, there is nothing in the derivation of HTI which requires that the distribution associated with each testimony be independent of the others conditional on the fact of the event in question. The need for this assumption is apparent only in the final version of MBL, which Merrill omits in its entirety.

Conversely, the final version of MBL requires that testimonies be independent of one another conditional on the fact of the event, *not* that they be unconditionally independent. In fact, for Equation (3) to apply it need not be the case that the veracities of the two witnesses are unconditionally independent. Merrill (1991, p. 99) seems to interpret Peirce as insisting that unconditional independence is necessary if MBL is to work. Though this condition is met by the numbers in Table 2, this is clearly *not* Peirce's contention. He says explicitly that, when it comes to witnesses offering true testimony "it is not necessary that the one should occur with the same proportionate frequency whether the other occurs or not, in general, without reference to whether the fact occurs or not" (CP 7.168).

18. Note that Kruskal (1988) emphasizes, long after Peirce, the difficulty with assuming the independence of testimonies in the context of Hume's argument.

19. Peirce does not elaborate on his reference to the "hideous wrongs" (Wiener, Peirce, and Langley 1947, p. 228) wrought by application of MBL in the judicial system, but Merrill (1991, pp. 108–111) offers what he takes to be a few plausible reasons why Peirce would have rejected the use the probability calculus in jurisprudence *in toto*. First among these is the supposed absurdity of the multiplicative rule for injunction: when two events are independent, the probability of their conjunction is the product of the probabilities of each event. He illustrates the absurdity by asking us to imagine "that the plaintiff in a civil case presents four independent arguments, each of which has a probability of .8 on the evidence." The plaintiff then would seem to have a strong case. "But the rule for conjunction tells us that the strength of the plaintiff's case as a whole is $.8^4 = .4096$." Since civil cases are decided by the preponderance of evidence, our plaintiff would lose on account of having too many reliable witnesses, which is absurd. The problem however, is not the conjunctive rule but a confused application of the probability calculus, at least as Peirce would see it. Whatever the merits of this argument, which Merrill borrows from Cohen (1977), it relies on exactly the error which leads Peirce to reject HTI. In simply multiplying the probability that each witness tells the truth, one computes only the probability that all are correct simultaneously (the numerator in HTI). What one *should* have computed is the probability that, given the testimony of these four witnesses, the events transpired as they say. Since this is precisely what Peirce points out in §3 of "The Logic of Drawing History," it is unlikely he would have endorsed Merrill's thesis. Indeed, from Equation (3) it is obvious that, if each witness is more likely to testify to a fact when it has occurred than when it has not, adding witnesses only increases the odds in favor of the fact having been as stated (only increases the strength of the plaintiff's case).