ROUNDABOUT THE RUNABOUT INFERENCE-TICKET

By J. T. Stevenson

IN HIS article "The Runabout Inference-Ticket" Professor A. N. Prior tries to show that there is an absurdity derivable from the theory "... that there are inferences whose validity arises solely from the meanings of certain expressions occurring in them."¹ For accounts of the theory other than his own Prior refers us to the writings of K. R. Popper, W. Kneale, P. F. Strawson, and R. M. Hare. I shall not be concerned whether he has accurately represented their versions of the theory (although I think this doubtful for example in the case of Popper), because Prior's interpretation is itself intrinsically interesting. Prior's argument strongly suggests that there is something wrong with the theory, as he presents it, but, unfortunately, he does not show us what is wrong with it. I wish to show (1) exactly where the theory, as stated by Prior, goes wrong, and (2) that the theory can be stated in such a way as to be quite sound.

According to the theory in question, the inference 'Grass is green and the sky is blue, therefore grass is green' is an analytically valid inference solely in virtue of the meaning of the word 'and'. The presumed analyticity of the inference is exhibited by the following statement of the meaning of the word 'and': "... (i) from any pair of statements P and Q we can infer the statement formed by joining P to Q by 'and' ... (ii) from any conjunctive statement P-and-Q we can infer P, and (iii) from P-and-Q we can always infer Q."²

Prior attempts to reduce the foregoing theory to absurdity by introducing a new connective 'tonk' and giving it a meaning in the way suggested by the theory. The complete meaning of 'tonk' is: "(i) from any statement P we can infer any statement formed by joining P to any statement Q by 'tonk' ... , and ... (ii) from any 'contonktive' statement P-tonk-Q we can infer the contained statement Q."³ He then shows that the following inference is valid in virtue of the meaning of 'tonk':

2 and 2 are 4.

Therefore, 2 and 2 are 4 tonk 2 and 2 are 5.

Therefore, 2 and 2 are 5.⁴

Prior does not say, but seems to imply, this: Since the theory allows to deduce, "in an analytically valid way", a patently false statement from a patently true one, there must be something radically wrong with the theory.

² Ibid.
³ Ibid.
⁴ Ibid.
In order to understand what has happened here, it is essential to notice that the theory requires us to give the meaning of logical connectives in terms of \textit{rules}. These rules are permissive: I take it that the force of 'we can infer', as it occurs in the foregoing definitions, is the same as 'we \textit{may} infer' or 'we are \textit{allowed} or \textit{permitted} to infer'. If 'we can infer' were taken to mean the same as 'we \textit{validly} infer', some of the things I shall say would have to be modified. But, in this case, if 'valid' were used in its ordinary sense (namely, such as to lead from truth only to truth and never to falsehood), Prior's definition of 'tonk' would become radically incoherent, indeed self-contradictory, and his argument trivially unsound and hence uninteresting. I shall take the more interesting and more usual interpretation that rules of inference are simply permissive. Granted this, we can now consider two important insights and one serious error in the theory.

The first insight concerns the meaning of logical connectives: the way in which we can express the meaning of connectives must be different from the way in which we express the meanings of non-logical words. In the first place, leaving aside Platonism, connectives are not used to denote, and hence the sort of semantical properties they have will be different from those of non-logical words. Second, logical terms are syncategorematic or incomplete symbols; they have no meaning in isolation. Since the most distinctive feature of a logical term is its syntactical properties, we can explain its meaning to someone unfamiliar with it by exhibiting how these syntactical properties affect the contexts in which the connective in question occurs. And a very convenient way to do this is to give the permissive rules governing the inferences we can make using the connective.

The second insight is that we ordinarily justify (i.e., validate) inferences by appealing to a permissive rule. If you question my inference 'If I don't leave in five minutes, I shall be late, and I am not going to leave in five minutes; so I shall be late', I justify it by appealing to the permissive rule \textit{modus ponens}.

The serious error in the theory consists in combining these two insights in an unfortunate way. It is assumed that we can \textit{completely} justify an inference by appealing to the meaning of a logical connective as stated in permissive rules. If this were so, we could, as Prior shows, justify any inference whatsoever by defining a logical connective in terms of permissive rules in such a way that we would be allowed to pass from true premises to a false conclusion.

The crucial point to be noted is this: in order to \textit{completely} justify an inference we must appeal to a \textit{sound} rule of inference. A complete justification of an inference has two parts: we must first \textit{validate} the inference by subsuming it under a rule, and secondly we must \textit{vindicate}
the rule itself by showing that it is a sound rule. A deductive rule is sound if and only if it permits only valid inferences, an inference being valid in this sense if and only if it is such that when the premises are true the conclusion must be true. The difficulty in our theory, then, is that it does not prevent us from defining connectives in terms of unsound permissive rules. Since no attempt is made to vindicate the rules used in the definitions, the definitions do not, by themselves, provide a complete justification of our inferences.

I now turn to the problem of stating the theory in such a way that it avoids the above difficulty. Basically, the theory states that certain inferences are completely justified solely in virtue of the meanings assigned to certain logical connectives. Since giving the meaning of a logical connective consists in giving its syntactical properties, we must show that, given a statement of the syntactical properties of a connective, the soundness of certain rules of inference can be demonstrated. There is no difficulty in doing this; it can be done, indeed, it has been done, for many different connectives, and there is no need to go into details here.

To be more precise, two qualifications should be made. First: the syntactical properties of a connective include both its formation and transformation properties, although here only its transformation properties are considered. Second: we can exhibit the transformation properties of a sentence connective in a calculus by making a value-table for it either so that the calculus remains uninterpreted, or so that it becomes interpreted. In the former case, we might use some arbitrary symbols for the values (say 0 and 1), and deal with pure syntactical properties. In the latter case, we use truth and falsity as values; and, since truth is a semantical notion, the calculus becomes to some extent interpreted, and we are no longer dealing with pure syntactical properties. For answering questions of soundness the latter method is the one which must be used; but for convenience I continue to speak simply of syntactical properties.

In a formal calculus we can state the syntactical properties of, say, a truth-functional binary sentence connective ‘o’, by stating, in the meta-language, the way in which the truth-value of the well-formed formula ‘poq’ is a function of (all possible combinations of) the truth-values of the components ‘p’ and ‘q’. We can then deduce from these statements, in a very rigorous way, a meta-theorem of the calculus (again stated in the meta-language) to the effect that such-and-such permissive rules are sound, i.e., lead from truths only to truths and never to falsehoods. Since from a statement of the meaning of a connective we can derive demonstrably sound permissive rules of inference governing that connective, the two qualifications are no obstacle.

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5 The distinction between validation and vindication is due to H. Feigl. See “De Principiis non Disputandum . . . ?” in Philosophical Analysis, ed. Max Black (Cornell University Press, 1950).

6 See any standard text, e.g., Church’s Introduction to Mathematical Logic.
connective, we may say that certain inferences are completely justified solely in virtue of the meanings of certain expressions occurring in them.

The important difference between the theory of analytic validity as it should be stated and as Prior stated it lies in the fact that he gives the meanings of connectives in terms of permissive rules, whereas they should be stated in terms of truth-function statements in a meta-language. The theory of analytic validity does not require that the meanings of connectives be given in terms of rules; as we have seen, to do so is to leave open the question of complete justification. What the correct theory of analytic validity does require is that the meanings of connectives be given in terms of statements of syntactical properties. When this is done the soundness of certain rules of inference is demonstrable, and thus inferences can be completely justified by appealing to the meanings of connectives. Using the latter method we block the introduction of a connective like Prior's 'tonk'. This can be seen as follows.

Consider these two truth-tables which exhibit in a graphic way the syntactical properties of two binary sentence connectives 'o' and '§'.

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<th></th>
<th>p</th>
<th>q</th>
<th>poq</th>
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<tbody>
<tr>
<td>A</td>
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<td>F</td>
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<tr>
<td>RA: p o' poq</td>
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<table>
<thead>
<tr>
<th></th>
<th>p</th>
<th>q</th>
<th>p§q</th>
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<tbody>
<tr>
<td>B</td>
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<td>F</td>
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<tr>
<td>RB: p§q o' q</td>
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</table>

From A it can be seen, intuitively, that the syntactical properties of 'o' permit us to demonstrate that the permissive rule RA, namely, from 'p' you may infer 'poq', is sound; and with a properly formulated statement of these syntactical properties it can be rigorously demonstrated to be sound. Similarly, from B it can be seen that the rule RB, namely, from 'p§q' you may infer 'q', is a sound rule. Prior's connective 'tonk' is governed by two rules like RA and RB. The syntactical properties of 'tonk', then, must be a combination of the syntactical properties of 'o' and '§'; and in order to demonstrate the joint soundness of the rules for 'tonk', we would have to construct a truth-table combining all the features of A and B. But it is obvious that this would involve ascribing contradictory syntactical properties to 'tonk'. This being so, it would be impossible to state consistently the meaning of 'tonk' in the manner I have suggested; and hence impossible to completely justify the inference from '2 and 2 are 4' to '2 and 2 are 5'. One could, of course, as Prior has done, state the meaning of 'tonk' in terms of rules and in this way justify (i.e., validate) '2 and 2 are 4, therefore, 2 and 2 are 5', but this would not completely justify the inference, for it would leave open the question as to the vindication of
the inference. And, of course, by definition it could never be vindicated, for it leads from an obvious truth to an obvious falsehood. I conclude, then, that there is nothing wrong with the theory of analytic validity when properly stated. 7

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PLAUSIBLE IMPLICATION

By Nicholas Rescher

I

I PROPOSE to examine the formal logic of an implication-relationship which has never, to my knowledge, been studied; a relationship which I shall designate as plausible implication. The conception which lies at the basis of this relationship is at once simple and natural. We shall say that a proposition p plausibly implies the proposition q if there is some proposition r which meets the conditions that: (i) r itself is likely (or probable), and (ii) the conjunction of p with r entails (strictly implies) q. In short, if ‘⇒’ represents entailment (strict implication) and ‘L’ represents the modality ‘is likely’ as applicable to propositions, then we may formally characterize plausible implication, to be symbolized as ‘⇒’, by the definition:

(D1.1) ‘p⇒q’=Df ‘(q⇒[(Lr) & ([p & r]⇒q)]’

In accordance with this definition, one proposition plausibly implies another when the conjunction of the former with some suitable proposition which is likely yields the latter as an entailed consequence.

To be sure, there are a great many ways of putting an exact construction on the common-language locution ‘If p, then quite likely q’.

The implication-relationship ‘⇒’ given by the definition (D1.1) clearly represents one, albeit only one, of these. It is this particular relationship which is here designated, for brevity and convenience, as ‘plausible implication’. Correspondingly, ‘p⇒q’ may be read as ‘If p then it is plausible that q’ or more compactly as ‘p plausibly implies q’. The present paper will be concerned to examine the formal logic of this

7 I should like to acknowledge the benefit I have had of a number of stimulating discussions with Wesley C. Salmon on this topic. He is not, of course, responsible for any errors I may have made.